Suggested Types of Problems
College Algebra

Systems of linear Equations and Inequalities

1. Solve the following systems of linear equations by substitution:
   a) (Easy) \[ 2x - y = 3 \]
      \[ x - 3y = 4 \]
   b) (Easy) \[ 4x - 5y = -7 \]
      \[ 3x + 8y = 30 \]
   c) (Medium) \[ \frac{1}{3}x - \frac{1}{4}y = 0 \]
      \[ -\frac{2}{3}x - \frac{3}{4}y = 2 \]

2. Solve the following systems of linear equations by using elimination:
   a) (Easy) \[ 2x + 5y = 5 \]
      \[ -4x - 10y = -10 \]
   b) (Easy) \[ 3x - 2y = 12 \]
      \[ 4x + 3y = 16 \]
   c) (Medium) \[ -0.5x + 0.3y = 0.8 \]
      \[ -1.5x + 0.9y = 2.4 \]

3. (Easy) Graph the system of equations to solve:
   a) \[ 2x + y = 3 \]
      \[ 2x + y = 7 \]
   b) \[ x - 2y = 1 \]
      \[ 2x - 4y = 2 \]

4. (Medium) **Health Club Management.** A fitness club has a budget of $915 to purchase two types of dumbbell sets. One set costs $30 each and the other deluxe set costs $45 each. The club wants to purchase 24 news sets of dumbbells. How many of each set should the club purchase?
5. **(Hard) Mixture.** In chemistry lab, Stephanie has to make a 37 milliliter solution that is 12% HCl. All that is in the lab is 8% and 15% HCl solutions. How much of each should she mix to get the desired solution?

6. Solve the following systems of equations:
   a) **(Easy)**
      \[-x + y - z = -1\]
      \[x - y - z = 3\]
      \[x + y - z = 9\]
   
   b) **(Medium)**
      \[3x + 2y + z = 4\]
      \[-4x - 3y - z = -15\]
      \[x - 2y + 3z = 12\]
   
   c) **(Hard)**
      \[x - z + y = 10\]
      \[2x - 3y + z = -11\]
      \[y - x + z = -10\]
   
   d) **(Medium)**
      \[2x_1 - x_2 + x_3 = 3\]
      \[x_1 - x_2 + x_3 = 2\]
      \[-2x_1 + 2x_2 - 2x_3 = -4\]

7. **(Hard)** Suppose you are going to eat only sandwiches for a week (seven days) for lunch and dinner (total of 14 meals). If your goal is a total of 4840 calories and 190 grams of fat, how many of each sandwich would you eat this week to obtain your goal? Consider the following table:

<table>
<thead>
<tr>
<th>Sandwich</th>
<th>Calories</th>
<th>Fat (Grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mediterranean Chicken</td>
<td>350</td>
<td>18</td>
</tr>
<tr>
<td>Tuna</td>
<td>430</td>
<td>19</td>
</tr>
<tr>
<td>Roast Beef</td>
<td>290</td>
<td>5</td>
</tr>
</tbody>
</table>

8. **(Hard)** Bob and Betty decide to place $20,000 of their savings into investments. They put some in a money market account earning 3% interest, some in a mutual fund that has averaged 7% a year, and some in stock that rose 10% last year. If they put $6,000 more in the money market than in the mutual fund, and the stocks and mutual fund have the same growth in the next year as in the previous year, they will earn $1,180 in the year. How much money did they put in each of the investments?

9. Find the form of the partial fraction decomposition. Do not solve for the constants:
   a) **(Easy)** \[\frac{3x+2}{x(x^2-25)}\]
   
   b) **(Medium)** \[\frac{3x+2}{x^2(x^2+25)^2}\]
   
   c) **(Medium)** \[\frac{x^2+2x-1}{x^4-9x^2}\]
10. Find the partial fraction decomposition for each rational function:

   a)  (Easy) \( \frac{1}{x(x+1)} \)

   b)  (Easy) \( \frac{9x-11}{(x-3)(x+5)} \)

   c)  (Medium) \( \frac{4x^2-7x-3}{(x+2)(x-1)^2} \)

   d)  (Medium) \( \frac{-2x^2-17x+11}{(x-7)(3x^2-7x+5)} \)

   e)  (Hard) \( \frac{5x+2}{x^3-8} \)

11. (Easy) Graph the linear inequalities:

   a)  \( y < 2x + 3 \)

   b)  \( 5x + 3y < 15 \)

   c)  \( 6x - 3y \geq 9 \)

12. Graph the system of inequalities or indicate that the system has no solution:

   a)  (Easy)

       \[
       \begin{align*}
       y & > 2x + 1 \\
       y & < 2x - 1
       \end{align*}
       \]

   b)  (Easy)

       \[
       \begin{align*}
       x + 2y & > 4 \\
       y & < 1 \\
       x & \geq 0
       \end{align*}
       \]

   c)  (Easy)

       \[
       \begin{align*}
       y & < x + 2 \\
       y & > x - 2 \\
       y & < -x + 2 \\
       y & > -x - 2
       \end{align*}
       \]

   d)  (Medium)

       \[
       \begin{align*}
       y + x & < 2 \\
       y + x & \geq 4 \\
       y & \geq -2 \\
       y & \leq 1
       \end{align*}
       \]
13. (Medium) Maximize \( z = 4x + 3y \) subject to:
\[
\begin{align*}
x &\geq 0, \\
y &\leq -x + 4, \\
y &\geq x.
\end{align*}
\]

14. (Hard) Minimize \( z = \frac{1}{3}x - \frac{2}{5}y \) subject to:
\[
\begin{align*}
x + y &\geq 6, \\
-x + y &\geq 4, \\
-x + y &\leq 6, \\
x + y &\leq 8.
\end{align*}
\]

15. (Hard) **Computer Business** A computer science major and a business major decide to start a small business that builds and sells a desktop and a laptop computer. They buy the parts, assemble them, load the operating system, and sell the computers to other students. The costs for parts, time to assemble the computer, and profit are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Desktop</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Parts</td>
<td>$700</td>
<td>$400</td>
</tr>
<tr>
<td>Assemble time(hours)</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Profit</td>
<td>$500</td>
<td>$300</td>
</tr>
</tbody>
</table>

They were able to get a small business loan in the amount of $10,000 to cover costs. They plan on making the computers over the summer and selling them at the beginning of the fall semester. They can dedicate at most 90 hours in assembling the computers. They estimate the demand for laptops will be at least three times the demand for desktops. How many of each type shall they make to maximize profit?

16. (Hard) **Production** A manufacturer of skis produces two models, a regular ski and a slalom ski. A set of regular skis give $25 profit and a set of slalom skis give a profit of $50. The manufacturer expects a customer demand of at least 200 pairs of regular skis and at least 80 pairs of slalom skis. The maximum number of pairs of skis that the manufacturer can produce is 400. How many of each model should be produced to maximize profits?

**Matrices**

1. (Easy) Determine the order of each matrix.
   a) \( \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \)
   b) \( \mathbf{B} = \begin{bmatrix} -1 & 3 & 6 & 8 \\
                                2 & 9 & 7 & 3 \\
                               5 & 4 & -2 & -10 \\
                               6 & 3 & 1 & 5 \end{bmatrix} \)
2. (Easy) Write the augmented matrix for each system of linear equations:
   
   a) 
   \[
   \begin{align*}
   x - y &= -4 \\
   y + z &= 3
   \end{align*}
   \]
   
   b) 
   \[
   \begin{align*}
   2x - 3y + 4z &= -3 \\
   -x + y - 2z &= 1 \\
   5x - 2y - 3z &= 7
   \end{align*}
   \]
   
3. (Easy) Write the system of linear equations represented by the augmented matrix
   
   \[
   \begin{bmatrix}
   3 & 0 & 5 & 1 \\
   0 & -4 & 7 & -3 \\
   2 & -1 & 0 & 8
   \end{bmatrix}
   \]
   
4. (Medium) Perform the indicated row operations on the augmented matrix
   
   \[
   \begin{bmatrix}
   1 & 0 & 5 & -10 \\
   0 & 1 & 2 & -3 \\
   0 & 2 & -3 & 0 \\
   0 & -3 & 2 & -1 
   \end{bmatrix}
   \]
   
   \[
   R_3 - 2R_2 \rightarrow R_3 \\
   R_4 + 3R_2 \rightarrow R_4
   \]
   
5. (Hard) Use row operations to transform the following matrix to reduced row-echelon form.
   
   \[
   \begin{bmatrix}
   -1 & 2 & 1 & -2 \\
   3 & -2 & 1 & 4 \\
   2 & -4 & 2 & 4 
   \end{bmatrix}
   \]
   
6. (Medium) Solve the system of linear equations using Gaussian elimination with back-substitution.
   
   \[
   \begin{align*}
   3x_1 + x_2 - x_3 &= 1 \\
   x_1 - x_2 + x_3 &= -3 \\
   2x_1 + x_2 + x_3 &= 0
   \end{align*}
   \]
   
7. (Hard) Solve the system of linear equations using Gauss-Jordan elimination.
   
   \[
   \begin{align*}
   x + 2y - z &= 6 \\
   2x - y + 3z &= -13 \\
   3x - 2y + 3z &= -16
   \end{align*}
   \]
   
8. Solve for the indicated variables.
   
   a) (Easy) 
   \[
   \begin{bmatrix}
   3 & 4 \\
   0 & 12
   \end{bmatrix}
   \begin{bmatrix}
   x - y \\
   4
   \end{bmatrix}
   =
   \begin{bmatrix}
   0 \\
   2y + x
   \end{bmatrix}
   \]
b) (Medium) \[
\begin{bmatrix}
9 & 2b + 1 \\
-5 & 16
\end{bmatrix} = \begin{bmatrix}
a^2 & 9 \\
2a + 1 & b^2
\end{bmatrix}
\]

9. (Easy) Given the matrices below perform the indicated operations for each expression, if possible.

\[A = \begin{bmatrix}
-1 & 3 & 0 \\
2 & 4 & 1
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & 2 & 1 \\
3 & -2 & 4
\end{bmatrix}, \quad C = \begin{bmatrix}
0 & 1 \\
2 & -1
\end{bmatrix}, \quad D = \begin{bmatrix}
2 & -3 \\
0 & 1
\end{bmatrix}\]

a) \(D - B\)
b) \(2B - 3A\)
c) \(-\frac{1}{5}C\)
d) \(C - A\)

10. Given the following matrices, perform the indicated operations for each expression, if possible.

\[A = \begin{bmatrix}
1 & 1 & -1 \\
0 & 3 & 1 \\
5 & 0 & -2
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & 2 & 1 \\
3 & -2 & 4
\end{bmatrix}, \quad C = \begin{bmatrix}
2 & 0 & -3 \\
3 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
-1 & 7 & 2 \\
3 & 0 & 1
\end{bmatrix}, \quad E = \begin{bmatrix}
-1 & 0 & 1 \\
2 & 1 & 4 \\
-3 & 1 & 5
\end{bmatrix}, \quad F = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}, \quad G = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\]

a) (Easy) \(GB\)
b) (Medium) \(B(A + E)\)
c) (Easy) \(CD + G\)
d) (Easy) \(FE - 2A\)

11. (Easy) Write the system of linear equations as a matrix equation.

a)
\[
\begin{align*}
3x + 5y - z &= 2 \\
x + 2z &= 17 \\
-x + y - z &= 4
\end{align*}
\]

b)
\[
\begin{align*}
x + y - 2z + w &= 11 \\
2x - y + 3z &= 17 \\
-x + 2y - 3z + 4w &= 12 \\
y + 4z + 6w &= 19
\end{align*}
\]

12. Determine whether \(B\) is the inverse of \(A\) using \(AA^{-1} = I\).

a) (Easy) \(A = \begin{bmatrix}
2 & 3 \\
1 & -1
\end{bmatrix}, \quad B = \begin{bmatrix}
\frac{1}{5} & \frac{3}{5} \\
\frac{1}{5} & \frac{-2}{5}
\end{bmatrix}\)
b) (Medium) \( A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \)

13. Find the inverse \( A^{-1} \).

a) (Medium) \( A = \begin{bmatrix} -2.3 & 1.1 \\ 4.6 & -3.2 \end{bmatrix} \)

b) (Hard) \( A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \)

14. Apply matrix algebra (use inverses) to solve the system of linear equations.

a) (Medium)
\[
\begin{align*}
\frac{2}{5}x + \frac{3}{7}y &= 1 \\
-\frac{1}{2}x - \frac{1}{3}y &= \frac{1}{6}
\end{align*}
\]

b) (Hard)
\[
\begin{align*}
x - y - z &= 0 \\
x + y - 3z &= 2 \\
3x - 5y + z &= 4
\end{align*}
\]

15. Calculate the determinant of each matrix.

a) (Easy) \( \begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix} \)

b) (Easy) \( \begin{bmatrix} -1.0 & 1.4 \\ 1.5 & -2.8 \end{bmatrix} \)

c) (Medium) \( \begin{bmatrix} 2 & 1 & -5 \\ 3 & -7 & 0 \\ -4 & -6 & 0 \end{bmatrix} \)

d) (Hard) \( \begin{bmatrix} 5 & -2 & -1 \\ 4 & -9 & -3 \\ 2 & 8 & -6 \end{bmatrix} \)

e) (Medium) \( \begin{bmatrix} \frac{3}{4} & -1 & 0 \\ 0 & \frac{1}{5} & -12 \\ 8 & 0 & -2 \end{bmatrix} \)
16. Use Cramer’s rule to solve each system of linear equations in two variables, if possible.

a) (Easy)

\[
\begin{align*}
3x - 2y &= -1 \\
5x + 4y &= -31
\end{align*}
\]

b) (Medium)

\[
\begin{align*}
\frac{2}{3}x + \frac{9}{4}y &= \frac{9}{8} \\
\frac{1}{6}x + \frac{1}{4}y &= \frac{1}{12}
\end{align*}
\]

17. Use Cramer’s rule to solve each system of linear equations in three variables.

a) (Medium)

\[
\begin{align*}
-x + y + z &= -4 \\
x + y - z &= 0 \\
x + y + z &= 2
\end{align*}
\]

b) (Hard)

\[
\begin{align*}
\frac{1}{2}x - 2y + 7z &= 25 \\
x + \frac{1}{4}y - 4z &= -2 \\
-4x + 5y &= -56
\end{align*}
\]

Sequences and Series

1. (Easy) Find the first four terms and the one hundredth term of the sequence given by \( a_n = \frac{(-1)^n}{(n+1)^2} \)

2. (Hard) Write an expression for the \( n \text{th} \) term of the sequence whose first few terms are 

\[
\frac{2}{3}, \quad \frac{4}{9}, \quad \frac{8}{27}, \quad \frac{16}{81}, \ldots
\]

3. (Medium) Find the first four partial sums and the \( n \text{th} \) partial sum of the sequence given by 

\( a_n = \frac{1}{n+1} - \frac{1}{n+2} \).

4. (Easy) Evaluate 

\[
\sum_{n=0}^{4} n^2
\]

5. (Medium) Write the sum \( \frac{2 \times 1}{1} + \frac{3 \times 2 \times 1}{1} + \frac{4 \times 3 \times 2 \times 1}{2 \times 1} + \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} + \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} \) using sigma notation.
6. (Medium) Write the first five terms of the recursively defined sequence defined by
\[ a_n = a_{n-1}a_{n-2}, \quad a_1 = 2, \quad a_2 = -3. \]

7. (Medium) Don takes a job out of college with a starting salary of $30,000. He expects to get a 3% raise each year. Write the recursive formula for a sequence that represents his salary \( n \) years on the job. Assume \( n = 0 \) represents his first year making $30,000.

8. (Easy) Find the first four terms of the sequence \( a_n = -3n + 5 \). Determine if the sequence is arithmetic, and if so find the common difference \( d \).

9. (Easy) Find the \( n^{th} \) term of the arithmetic sequence given the first term \( a_1 = 5 \) and the common difference \( d = -\frac{3}{4} \).

10. (Medium) Find the first term, \( a_1 \), and the common difference, \( d \), of the arithmetic sequence whose 5\(^{th} \) term is 44, and whose 17\(^{th} \) term is 152.

11. (Easy/Medium) Find the 100\(^{th} \) term of the arithmetic sequence \{9, 2, -5, -12, \ldots \}.

12. (Medium) Find the sum
\[ \sum_{n=1}^{30} (-2n + 5) \]

13. (Medium) Find \( S_{43} \), the 43\(^{rd} \) partial sum of the arithmetic sequence \{1, \frac{1}{2}, 0, -\frac{1}{2}, \ldots \}.

14. (Medium) An amphitheater has 40 rows of seating with 30 seats in the first row, 32 in the second row, 34 in the third row, and so on. Find the total number of seats in the amphitheater.

15. (Medium) How many terms of the arithmetic sequence \{5, 7, 9, \ldots \} must be added to get 572?

16. (Easy) Determine if the sequence \{2, -10, 50, -250, 1250, \ldots \} could be geometric, and if so find the common ratio \( r \).

17. (Easy) Find the eighth term of the geometric sequence \{5, 15, 45, \ldots \}.

18. (Hard) Find the fifth term of the geometric sequence given that the third term is \( \frac{63}{4} \) and the sixth term is \( \frac{1701}{32} \).
19. (Medium) Find $S_5$, the fifth partial sum of the geometric sequence $\{1, 0.7, 0.49, 0.343, \ldots \}$

20. (Easy) Evaluate

\[ \sum_{k=1}^{5} \left( -\frac{2}{3} \right)^k \]

21. (Easy) Find the sum of the infinite geometric series $2 - \frac{2}{5} + \frac{2}{25} - \frac{2}{125} \cdots$

22. (Medium) Write $0.\overline{321}$ in reduced fraction form.

23. (Medium) Expand $(2 - 3x)^5$ using Pascal’s triangle.

24. (Easy) Calculate the binomial coefficient \( \binom{20}{3} \)

25. (Medium) Find the term that contains $x^3$ in the expansion of $(y - 3x)^{10}$.

26. (Hard) Find the middle term of the expansion $(x^2 + 1)^{18}$.

27. (Medium) Find the coefficient of the simplified third term in the expansion of \( (\sqrt{2} + y)^{12} \)