# **Review: Math 3300: Foundations of Abstract Algebra**

**Goal:** To review the material traditionally covered in Foundations of Abstract Algebra course (Math 3300).

**Structure of the Module:** The module is divided in 10 parts (five in <u>Part 1</u> and five in <u>Part 2</u> of this module). Each of the parts has a soft deadline attached to it with aim to guide students in achieving the objectives and reviewing the material in a timely manner.

Each part (except the last one <u>Practice - GRE Problems</u>) starts with a list of objectives that need to be achieved after reviewing a particular topic (listed in **Topics Covered**) and has two assignments attached to it.

**Topics Covered:** 

Part 1 (Week 1)

Day 1: Preliminaries (review of Integers and Equivalent Relations)

Day 2: Introductions of Groups

Day3: Groups

Day 4: Finite Groups and Subgroups

Day 5: Cyclic Groups and Permutations Groups

Part 2 (Week 2)

Day 1: Cosets and Lagrange's Theorem

Day 2: Group Homomorphisms and Isomorphisms

Day 3: Normal Subgroups and Factor Groups

Day 4: Introduction to Ring Theory

Day 5: Practice - GRE Problems

**Recommended Textbook:** The topics listed above follow closely the material covered in Chapters 0 -10 (excluding Chapter 8) in Gallian's book:

J. Gallian, Abstract Algebra, Contemporary Abstract Algebra, 8th Edition, by Joseph A. Gallian

(Note that any edition of this book will cover the same material)

Additional Reference Textbooks (can be found in the Library): The topics listed above are topics covered in any traditional Abstract Algebra book. Attached is a list of books that can be found in the UVU Library.

## **Integers and Equivalent Relations**

# **Chapter 0: Preliminaries (Integers and Equivalence Relations)**

Upon successful review of Chapter 0 (Gallian's textbook), student(s) should be able to

- define and use/apply the following concepts: a divisor and a multiple of an integer, a prime integer, greatest common divisor (gcd), relatively prime integers, least common multiple (*lcm*)

- state and apply the Well Ordering Principle, the Division Algorithm, properties of *gcd*, Euclid's Lemma, and the Fundamental Theorem of Arithmetic

- work with modular arithmetic

- work with complex numbers and use/apply their properties

- define and use/apply the following concepts: equivalence relation, partition, equivalence classes

- define and use/apply the following concepts: a function (mapping), composition of functions, a one-to-one function, an onto function, and be familiar with their properties.

**Introduction to Groups** 

## **Chapter 1: Introduction to Groups**

Upon successful review of Chapter 1 (Gallian's textbook), student(s) should be able to

- define, identify, and work with the Dihedral groups
- construct a Cayley table of finite groups

Groups

## **Chapter 2: Groups**

Upon successful review of Chapter 2 (Gallian's textbook), student(s) should be able to

- state definitions of a binary system and a group

- prove that a set is a binary system/group under a given operation

- recognize, prove, and work with variety of groups (Z, R, Q, C,  $Z_n$ , GL(2,R), SL(2,R), U(n), etc.)

- prove various elementary properties of groups (uniqueness of identity and inverses, socks-shoes property, solving equations, etc.)

- use elementary group properties to prove more advanced group results

## Finite Groups and Subgroups

## Chapter 3: Finite Subgroups; Subgroups

Upon successful review of Chapter 3 (Gallian's textbook), student(s) should be able to

- state the definition of an order of a group/element, subgroup, center of a group, centralizer of an element in a group

- apply these definitions on a concrete examples
- prove group results using these concepts
- state and use Subgroup Tests in various settings

- prove, find, and work with various examples of subgroups (cyclic subgroup, center of a group, centralizer of an element in a group

#### **Cyclic Groups and Permutations Groups**

## **Chapter 4: Cyclic Groups**

Upon successful review of Chapter 4 (Gallian's textbook), student(s) should be able to

- work with cyclic groups (know examples of cyclic groups, prove that a group is cyclic, find which of the groups considered in Chapter 2 are cyclic, etc.)

- find generators of a cyclic group, and in particulars generators of  $Z_n$
- familiar with various results about orders of elements in a cyclic group
- state and use the Fundamental Theorem of Cyclic Groups
- (optional) use the \phi function and Subgroup lattice of  $Z_n$  groups

#### **Chapter 5: Permutation Groups**

Upon successful review of Chapter 4 (Gallian's textbook), student(s) should be able to

- state and use/apply definitions of a permutation of a set A, permutation group of A, symmetric groups of degree n, and alternating group of degree n

- use cyclic notation of permutations

- work with various properties of permutations (product of disjoint cycles, disjoint cycles commute, order of a permutation, even/odd permutation

## **Cosets and Lagrange's Theorem**

# Chapter 7: Cosets and Lagrange's Theorem

Upon successful review of Chapter 7 (Gallian's textbook), student(s) should be able to

- state, apply, and use definition of a coset of a subgroup in a group
- find cosets of various subgroups
- prove basic coset properties
- use basic properties of cosets to prove more advanced group results
- state, prove, and use/apply Lagrange's theorem and its Corollaries
- state and use the Classification of groups of order 2p

**Group Homomorphisms and Isomorphisms** 

# Chapter 6: Group Isomorphisms

Upon successful review of Chapter 6 (Gallian's textbook), student(s) should be able to

- state and apply the definition of group isomorphisms, automorphisms, inner automorphisms induced by an element a
- show that a map is an isomorphism, automorphism, inner automorphism
- prove that two groups are isomorphic
- state, prove, and apply the Cayley's Theorem
- state, prove, and use/apply various isomorphism properties
- define and find the Automorphism group and Inner Automorphism group

## Chapter 10: Group Homomorphism

Upon successful review of Chapter 10 (Gallian's textbook), student(s) should be able to

- state and apply the definition of group homomorphisms, kernel of a homomorphism
- show that a map is an homomorphism and find its kernel
- state, prove, and use/apply various homomorphism properties
- state, prove, and use/apply the First Isomorphism Theorem and its Corollaries

## **Normal Subgroups and Factor Groups**

## **Chapter 9: Normal Subgroups and Factor Groups**

Upon successful review of Chapter 9 (Gallian's textbook), student(s) should be able to

- state and apply the definition of a normal subgroup, a factor group

- prove that a subgroup is normal using the definition as well as the Normal Subgroups Test

- form factor groups in various group setting

- prove various applications of Factor Groups (G/Z Theorem, Cauchy's Theorem for Abelian Groups)

**Introduction to Ring Theory** 

# Chapter 12, 13: Introductions to Rings, Integral Domains, and Fields

Upon successful review of Chapter12 (Gallian's textbook), student(s) should be able to

- state and use/apply the definition of a ring, unit, unity, zero divisor, integral domain, and field

- provide various examples of rings
- state and prove various elementary properties of rings