State Math Contest 2025 Junior Level

Instructions:

- Calculators, cell phones and other computational devices are not permitted (you can only use pens, pencils and paper to work on your answers, and then mark your answers with a number two pencil on the answer sheet).
- Correct answers are worth 5 points. Unanswered questions will be given 1 point. Incorrect answers will be worth 0 points. *This means that it will not, on average, increase your score to guess answers randomly.*
- Fill in the answers on the answer sheet using a number two pencil.
- Time limit: 120 minutes.
- When you are finished, please give the exam and any scratch paper to the test administrator.
- Good luck!

---

1. Find $8^{(\log_2 10)(\log_{10} 5)}$.

   A. $\log_{10} 125$
   B. 5
   C. 32
   D. 125
   E. 8^5

2. A class contains ten people. From this class, a president and vice president must be selected, followed by the appointment of a group of three assistants to help them. How many ways can the president, vice president and the group of three assistants be selected?

   A. 5040
   B. 252
   C. 5400
   D. 720
   E. 840

3. A game is played where three people, players A, B, and C, take turns flipping a fair coin, with A going first, then B, and then C, and then A again, and so on until someone wins. The first one to flip heads wins. Find the probability that player A will win.

   A. $\frac{1}{3}$
   B. $\frac{1}{2}$
   C. $\frac{5}{8}$
   D. $\frac{4}{7}$
   E. $\frac{2}{3}$

4. There are a hundred students in at least one of three clubs, chess club, math club and robotics club. There are 46 in chess club, 48 in math club and 54 in robotics club. There are 20 in both chess and math clubs, 18 in both robotics club and math club, and 22 in both chess club and robotics club. If a student is randomly selected from the math club, what is the probability that this student is also in both of the chess and the robotics clubs?

   A. $\frac{1}{3}$

   B. $\frac{1}{6}$

   C. $\frac{2}{3}$

   D. $\frac{5}{8}$

   E. $\frac{1}{4}$

5. Let S be the smallest set of positive integers such that
   (a) 22 is in S,
   (b) if n^2 is in S, then n is in S, and
   (c) if n is in S, then $(n+3)^2$ is in S.
   What is the sum of the smallest two positive integers not in S?

   A. 3

   B. 4

   C. 5

   D. 6

   E. It cannot be determined.

6. Clark runs a 26 mile marathon. One-third the distance is uphill, one-third is flat and the other one-third is downhill. She runs uphill at a constant rate of 4 mph, flat at a constant rate of 6 mph, and downhill at a constant rate of 9 mph. What is her average speed for the entire marathon?

   A. $\frac{108}{19}$

   B. $\frac{19}{3}$

   C. $\frac{113}{20}$

   D. $\frac{95}{13}$

   E. 7

7. A regular hexagon and a square have the same perimeters. If you divide the area of the hexagon by the area of the square, what number do you get?

   A. $\frac{3}{2}$

   B. $2\sqrt{3}$

   C. $\frac{4\sqrt{3}}{3}$

   D. 2

   E. $\frac{2\sqrt{3}}{3}$

8. Let $\cos(x) - \sin(x) = \frac{\sqrt{11}}{4}$. Find $|\cos(2x)|$.

   A. $\frac{9}{16}$

   B. $\frac{\sqrt{231}}{16}$

   C. $\frac{\sqrt{5}}{16}$

   D. $\frac{5}{16}$

   E. $\frac{3\sqrt{23}}{16}$

9. If $i^2 = -1$, find i^i.

   A. $e^{-\frac{\pi}{2}}$

   B. 1

   C. -1

   D. $e^{\pi i}$

   E. $e^{\frac{\pi}{2} i}$

10. You have \$6. Your friend offers you a betting game where you roll a pair of fair six-sided dice once or twice according to the following rules.

- After the first roll of two dice, if the sum of the two dice is more then 6, then the bet ends and your friend will pay you an amount in dollars equal to the sum. (For example: if you roll a 4 and a 3 in the first roll, your friend will pay you \$7.)
- If the sum of the two dice in the first roll is not greater than 6, then you roll the two dice again. If the sum of the two dice in the second roll plus the sum of the two dice in the first roll is greater than 6, then your friend will pay you the total sum of these two sums in dollars. Otherwise, you pay your friend the total sum of these two sums in dollars.

What is the probability that your friend pays you after two rolls?

A. $\dfrac{175}{432}$

B. $\dfrac{329}{432}$

C. $\dfrac{419}{432}$

D. $\dfrac{373}{648}$

E. $\dfrac{419}{648}$

11. A traditional rotary combination lock uses a sequence of three numbers from 0 to 39 as the "combination" that is needed to unlock the lock. Assume repeating numbers are allowed for this type of lock. For example, 23-9-23, 1-2-3, 21-21-21, and 34-2-15 are all viable combinations for this type of lock. How many possible combinations for this type of lock contain at least one number that is a multiple of 3 in them?

A. $17,304$

B. $22,440$

C. $46,424$

D. $51,912$

E. $67,200$

12. In an acute (all three angles are less than $90°$) isosceles triangle, the smallest angle is half of a largest angle. What is the ratio of the shortest side to a longest side?

A. $\dfrac{1+\sqrt{5}}{2}$

B. $\dfrac{1}{2}$

C. $\dfrac{\sqrt{3}-1}{2}$

D. $\dfrac{\sqrt{5}-1}{2}$

E. $\dfrac{1+\sqrt{3}}{4}$

13. Find the sum $\sum_{n=1}^{2025} \cos\left(\frac{n\pi}{6}\right)$.

A. $-\frac{1+\sqrt{3}}{2}$

B. -1

C. $\frac{1+\sqrt{3}}{2}$

D. 0

E. $-\frac{3+\sqrt{3}}{2}$

14. Let

$$x = 3 + \cfrac{8}{4 + \cfrac{8}{4 + \cfrac{8}{4 + \cfrac{8}{\ddots}}}}$$

Which of the following is equal to x?

A. $3+\sqrt{2}$

B. $1+2\sqrt{2}$

C. $1+2\sqrt{3}$

D. $4\frac{1}{3}$

E. 4

15. A cube of edge length 12 is cut into two solids by a plane passing through four points (I, J, A and C) where I and J are midpoints along the edges EH and HG respectively, and A, C are vertices of the cube as shown in the picture. What is the total surface area of the smaller solid?

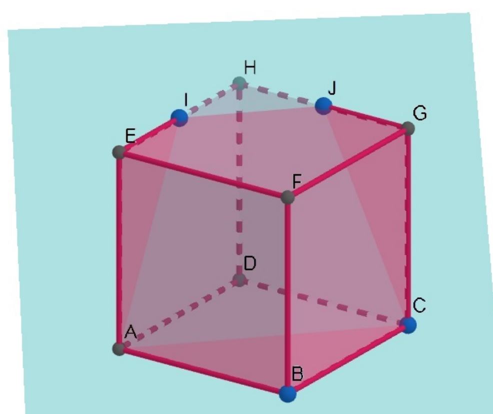
A. 388

B. 400

C. 432

D. 468

E. 502



16. An NBA player makes 80% of his free throws. If he shoots a sequence of 6 free throws, what is the probability that he makes at least 4 of them?

    A. $\dfrac{4^4 \cdot 21}{5^6}$

    B. $\dfrac{4^4 \cdot 11}{5^5}$

    C. $\dfrac{4^5 \cdot 13}{5^6}$

    D. $\dfrac{4^4 \cdot 21}{5^5}$

    E. $\dfrac{4^5 \cdot 3}{5^5}$

17. In the figure, $\overline{AC}$ is a diameter of the circle $ABCD$. $\overline{PB}$ and $\overline{PD}$ are tangents to the circle. When you extend $\overline{AD}$ and $\overline{BC}$, the two lines meet at Q. If $\angle BPD = 68^\circ$, then $\angle AQB =$

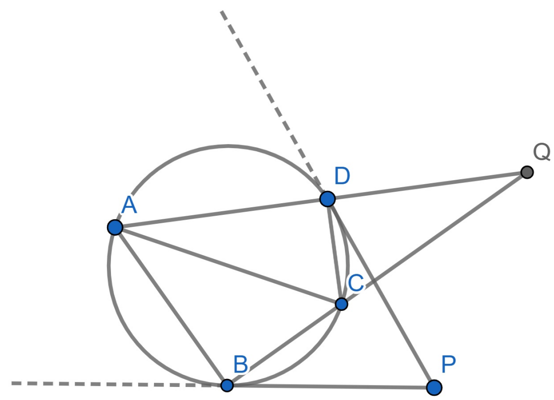
    A. 22°

    B. 28°

    C. 32°

    D. 34°

    E. 36°



18. In the multiplication problem shown below, a, b, c, and d each represents a distinct digit from 0 to 9.

$$\begin{array}{r} bc \\ \times \quad bc \\ \hline dc \\ bc \; \\ \hline aac \end{array}$$

What is the result of the following operations: $(b \cdot a) + (d \cdot c) - (a \cdot d) =?$

    A. 20

    B. 21

    C. 23

    D. 24

    E. 26

19. In the base 12 numeration system, the allowed digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E from the smallest to largest. Perform the subtraction: $ET5_{(twelve)} - TET_{(twelve)}$

A. $T7_{(twelve)}$

B. $E4_{(twelve)}$

C. $87_{(twelve)}$

D. $139_{(twelve)}$

E. $T3_{(twelve)}$

20. Let k be a positive integer. Let S_k be the product of 20 consecutive positive multiples of k starting from k. Find $\dfrac{S_{4k} + S_{2k}}{S_k}$. (Note: $n! = n(n-1)(n-2)\cdots 1$ for a positive integer n and $0! = 1$)

A. $\dfrac{(4k)! + (2k)!}{k!}$

B. $(20!)(4^{20} + 2^{20})$

C. 6

D. $4! + 2!$

E. $2^{20}(2^{20} + 1)$

21. If B and C are sets (collections of objects), let $|B|$ denote the number of objects in set B and $B \cap C$ denote the intersection of B and C (the collection of common objects in B and C). For a positive integer k, define $A_k = \{x \mid k < x < k+5, x \text{ is positive integer}\}$, i.e. the set of positive integers between k and $k+5$. Find the product of the t values satisfying $|A_{26} \cap A_t \cap A_{t+1}| = 1$.

A. 594

B. 600

C. 624

D. 644

E. 756

22. In triangle $\triangle ABC$, points D, E, and F lie on sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively, such that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ meet at point P. Given that $\dfrac{BD}{DC} = 2$, $\dfrac{CE}{EA} = 3$, and the area of triangle $\triangle ABC$ is 120 square units. Find the area of triangle $\triangle CBP$.

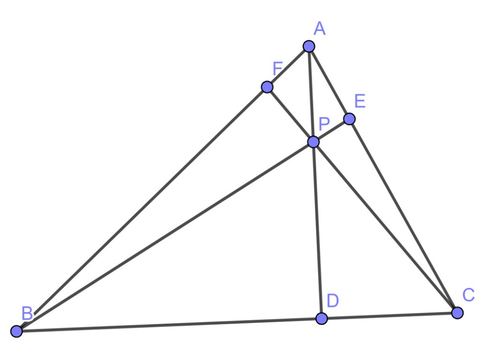
A. 24

B. 40

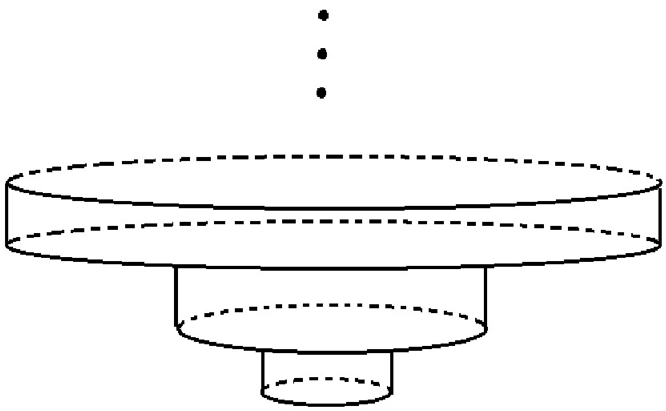
C. 60

D. 80

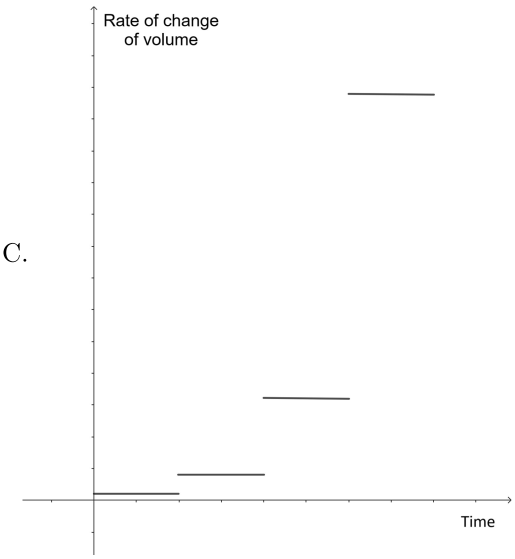
E. 100



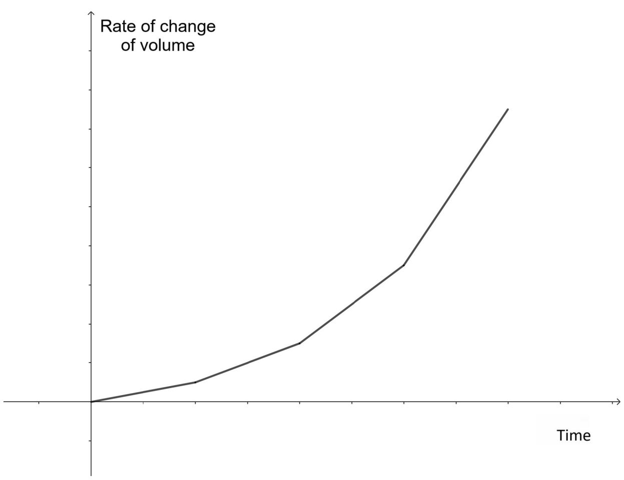
23. A bowl is constructed in layers of equal height, H, with each layer's circular base having a radius that doubles as you move up one layer. Specifically, the bottommost layer has a base radius of r, the second layer has a base radius of $2r$, the third layer has a base radius of $4r$, and so on. The bowl is being filled with water while standing upright. Let h be the height of water inside the bowl at any time t. Suppose that the rate of change of h with respect to time is constant throughout the whole time. Which of the following graphs best represents the rate of change of volume of water inside the bowl with respect to time as the bowl fills?



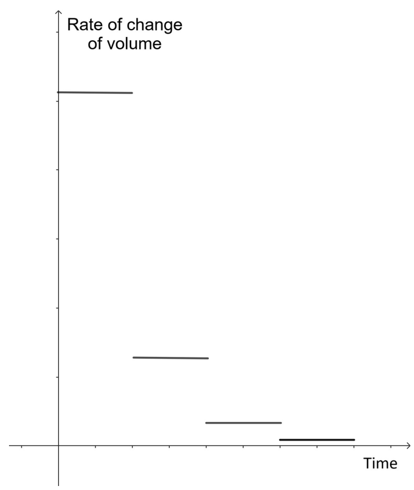
C.



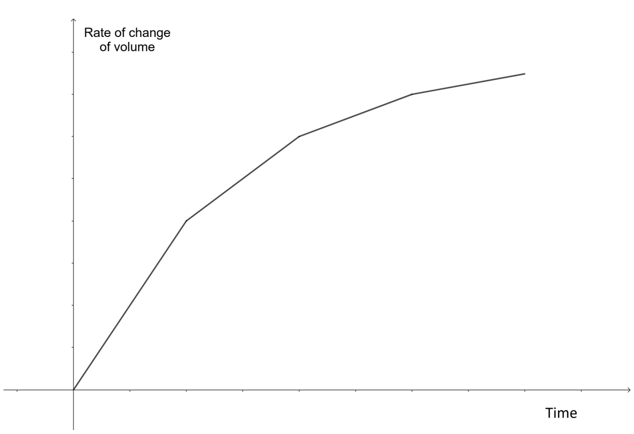
A.



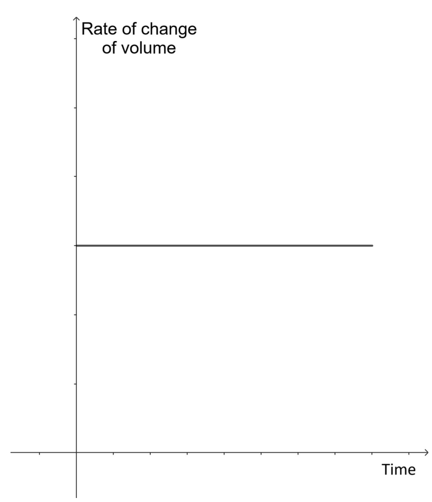
D.



B.



E.



24. The following dialogues feature four close friends talking about whether they would go to an amusement park for spring break. It is known that their statements are accurate. Only two of them actually went. Who are they?

Yuna: I will go if Thomas goes.
Thomas: I will go if Celina goes.
Hyrum: I will go if Yuna goes.
Celina: I'm not sure whether I'm going.

A. Yuna and Thomas

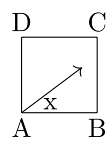
B. Yuna and Hyrum

C. Thomas and Celina

D. Thomas and Hyrum

E. Yuna and Celina

25. A light has been launched from the vertex A of a square below. The light reflects off the sides until it hits a vertex. When a ray of light strikes a surface, the angle of incidence equals the angle of reflection. Which vertex will the light reach if $\tan x = \dfrac{3}{4}$?



A. A

B. B

C. C

D. D

E. The light never reaches a vertex.

26. What are the last two digits (tens and ones digits) of $7^{2025} \times 11^{2024} \times 13^{2023} \times 17^{2022}$?

A. 53

B. 67

C. 71

D. 83

E. 97

27. For any real numbers x and y, define $x \circledast y = \dfrac{2xy}{3(x-y)}$. If a and b are positive integers such that $a \circledast b = 20$, how many possible values of a are there?

A. 9

B. 10

C. 12

D. 13

E. Infinitely many

28. Suppose $f(x) = -x - 1$ for $-3 \leq x < 0$, $f(3) = 0$ and satisfies $xf(x+3) = f(x) + 1$ for all real numbers x. What is the explicit formula for $f(x)$ for $6 \leq x < 9$?

A. $f(x) = 0$

B. $f(x) = 1$

C. $f(x) = x + 1$

D. $f(x) = \dfrac{1}{x-3}$

E. $f(x) = \dfrac{1}{x+3}$

29. Let $z_n = \left(\dfrac{1+i}{2}\right)^n$ where $i^2 = -1$. What is z^{50}?

A. $\dfrac{i}{2^{25}}$

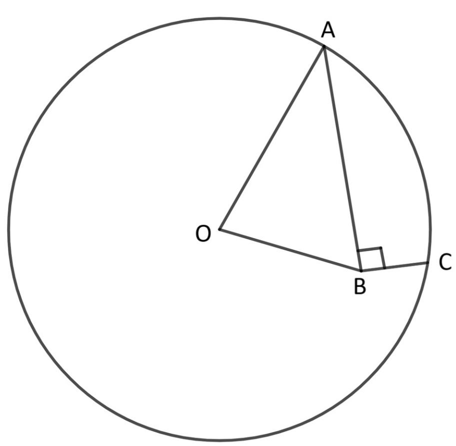
B. $\dfrac{1+i}{2^{25}}$

C. $\dfrac{1-i}{2^{25}}$

D. $\dfrac{1}{2^{50}}$

E. $-\dfrac{1+i}{2^{50}}$

30. A and C lie on a circle center O with radius $\sqrt{50}$. The point B inside the circle is such that $\angle ABC = 90°$, $AB = 6, BC = 2$. Find OB.



A. $\sqrt{26}$

B. $\sqrt{30}$

C. $\sqrt{50}$

D. $\sqrt{55}$

E. $\sqrt{60}$