State Math Contest 2026 Junior Level

Instructions:

- Calculators, cell phones and other computational devices are not permitted (you can only use pens, pencils and paper to work on your answers, and then mark your answers with a number two pencil on the answer sheet).
- Correct answers are worth 5 points each. Unanswered questions are worth 1 point each. Incorrect answers are worth 0 points each. *This means that it will not, on average, increase your score to guess answers randomly.*
- Fill in the answers on the answer sheet using a number two pencil.
- Time limit: 120 minutes.
- When you are finished, please give the exam and any scratch paper to the test administrator.
- Good luck!

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1. What is the remainder when $x^{2026} + 2026$ is divided by $x + 1$?
   A. 2027
   B. -1
   C. 2025
   D. 0
   E. 1

2. Find $\sqrt{999^2 + 999 + 1000}$.
   A. 999
   B. 1000
   C. 1001
   D. $\sqrt{999000}$
   E. 10^6

3. How many integer solution pairs are there for the equation $xy - 2x - 3y + 6 = 0$, where $0 < x, y < 10$?
   A. 1
   B. 9
   C. 17
   D. 18
   E. 81

4. Which numbers x satisfy the equation $(\log_2 x)(\log_x 5) = \log_2 5$?

   A. All real numbers x.

   B. The numbers $x = 5$ and $x = 2$ only.

   C. This is not true for any real number x.

   D. All positive real numbers.

   E. All positive real numbers other than 1.

5. A rectangular garden has a length that is 3 meters more than twice its width. If the area of the garden is 90 square meters, find the perimeter of the garden.

   A. 36

   B. 39

   C. 42

   D. 45

   E. 48

6. How many different arrangements are there when placing two identical squares, four identical circles, and one triangle in a line?

   A. 210

   B. 105

   C. 140

   D. 35

   E. 5040

7. Hoses A, B, C, and D are being used to fill a single pool. If hose A was the only one being used, it would take 8 hours to fill the pool. With hose B only it would take 6 hours; with hose C only it would take 4 hours; and with hose D only it would take 3 hours. How long does it take to fill the pool if all four hoses are being used at once?

   A. 2 hours

   B. 1.5 hours

   C. $\frac{7}{6}$ hours

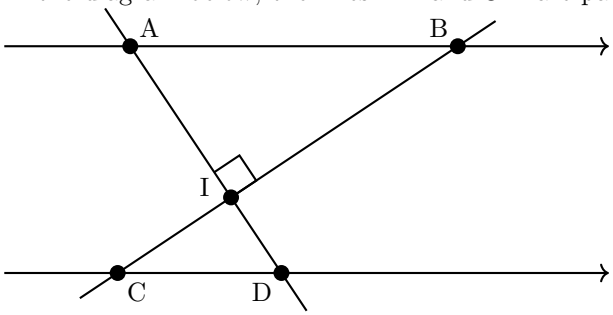
   D. $\frac{8}{7}$ hours

   E. 3 hours

8. When rolling a fair six-sided die, what is the probability of rolling at least one number that is a multiple of 3 in 6 rolls?

   A. $\frac{1}{3}$
   B. $\frac{5}{6}$
   C. $\frac{11}{36}$
   D. $\frac{64}{729}$
   E. $\frac{665}{729}$

9. In the diagram below, the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel. Let $x = AB$, $y = CD$, and $z = BI$.



   Which of the following is an expression whose value is equal to DI?

   A. $x\sqrt{y^2 - z^2}$
   B. $\frac{y}{x}\sqrt{x^2 - z^2}$
   C. $\frac{x}{z}\sqrt{y^2 - x^2}$
   D. $\frac{x}{y}\sqrt{y^2 - z^2}$
   E. $y\sqrt{x^2 - z^2}$

10. Let A be the set of even non-negative integers which are less than or equal to 2026. Suppose a number is randomly chosen from A. What is the probability that this number is divisible by both 4 and 6?

    A. $\frac{168}{1013}$
    B. $\frac{169}{1013}$
    C. $\frac{28}{169}$
    D. $\frac{1}{6}$
    E. $\frac{85}{507}$

11. Find the sum of the digits of the greatest common divisor of 128,520 and 12,393.

A. 17

B. 89

C. 26

D. 20

E. 18

12. Let $n = 2026$. Find the remainder when $1 + n + n^2 + n^3 + \cdots + n^{67}$ is divided by 7.

A. 0

B. 1

C. 2

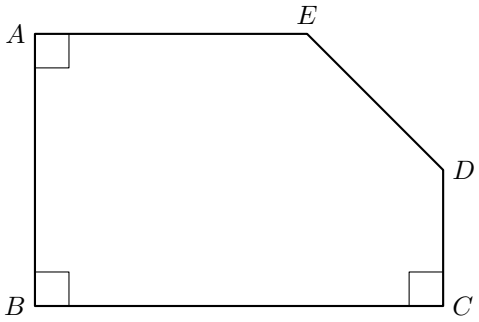
D. 3

E. 4

13. Pentagon $ABCDE$ lies in a plane and has right angles at vertices A, B, and C. The side lengths are given by

$$AB = 3x + 10, \qquad BC = 4x + 8, \qquad CD = 10, \qquad AE = 2x + 8,$$

and the segment ED has length $2\sqrt{13}$. Find the perimeter of pentagon $ABCDE$.



A. $54 + 2\sqrt{13}$

B. $63 + 2\sqrt{13}$

C. $72 + 2\sqrt{13}$

D. $63 + 4\sqrt{13}$

E. $72 + 2\sqrt{39}$

14. A particle moves along a line. It first moves forward $\frac{2}{3}$ units. Each subsequent move is $\frac{2}{3}$ of the length of the previous move but in the opposite direction. What is the particle's final position relative to its starting point?

A. $\frac{2}{5}$
B. $\frac{2}{3}$
C. $\frac{4}{5}$
D. $\frac{1}{5}$
E. 0

15. Let $f(x)$ be a positive function so that $f(x+y) = f(x)f(y)$ for all real x and y. Consider the following statements:

(1) $f(0) = 1$.

(2) $f(-1) = \frac{1}{f(1)}$.

(3) $f(2) = f(1)^2$.

Which of the following must be true?

A. Only statement (1) must be true.
B. Only statement (2) must be true.
C. Only statement (3) must be true.
D. Only statements (2) and (3) must be true.
E. Statements (1), (2), and (3) must all be true.

16. For a given unfair coin, the probabilities of obtaining heads and tails on a single flip are $\frac{1}{4}$ and $\frac{3}{4}$, respectively. What is the probability of obtaining exactly one or exactly two heads when the coin is flipped 5 times?

A. $\frac{243}{1024}$
B. $\frac{405}{1024}$
C. $\frac{675}{1024}$
D. $\frac{405}{512}$
E. $\frac{5}{16}$

17. Consider the system of equations

$$ax + 4y = 7$$
$$3x + by = 9.$$

For how many pairs of positive integers (a, b) does this system have **no** solution?

A. 12
B. 6
C. 4
D. 3
E. 2

18. Let $z = x + yi$ be a complex number. Solve

$$|z - 2| = |z + 2i|.$$

Which of the following describes the solution set in the xy-plane?

A. $\{\, x + yi \mid y = x \,\}$
B. $\{\, x + yi \mid y = -x \,\}$
C. $\{\, x + yi \mid x = 0 \,\}$
D. $\{\, x + yi \mid y = 0 \,\}$
E. $\{\, x + yi \mid y - x = 2 \,\}$

19. How many 5-digit numbers have strictly increasing digits?

A. 126
B. 252
C. 84
D. 210
E. 120

20. A positive integer k which is less than a thousand is selected at random (so each integer has an equal probability of being chosen). Find the probability that $\log_2 k + \log_2 2k$ is an integer.

A. $\frac{499}{999}$

B. $\frac{10}{999}$

C. $\frac{2}{333}$

D. $\frac{3}{500}$

E. 0

21. Let $f(x)$ be a periodic function with a period of 4. On the interval $[0, 2]$, the function is defined as $f(x) = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$. If $f(x)$ is also even, find the value of $f(11) + f(2026)$.

A. 0

B. 1

C. -1

D. $\sqrt{2}$

E. $-\sqrt{2}$

22. Let s be the sum of all real solutions to the equation

$$2^x = x^2.$$

Which of the following intervals contains s?

A. $[2, 3)$

B. $[3, 4)$

C. $[4, 5)$

D. $[5, 6)$

E. $[6, 7)$

23. Find $(f \circ g \circ h)(24)$ given that

$$f^{-1}(x) = 8^x, \qquad g^{-1}(x) = \log_2 \sqrt{x}, \qquad h^{-1}(n) = n!,$$

for integers $n \geq 1$.

A. $\frac{3}{8}$
B. $\frac{4}{3}$
C. $\frac{8}{3}$
D. 3
E. 8

24. Two observers are standing on level ground, 400 feet apart. The angle of elevation of a balloon from the first observer is $\frac{\pi}{3}$, and the angle of elevation of the balloon from a second observer is $\frac{\pi}{4}$, where each angle of elevation is measured from the horizontal line through the observer's eyes to the line of sight to the balloon. The balloon lies in the same vertical plane as the two observers and its vertical projection onto the ground lies between them. Assume both observers' eyes are at the same height above the ground. Find the straight-line distance from the first observer's eye to the balloon.

A. $400\sqrt{2}$
B. $\frac{400}{\sqrt{2}}$
C. $\frac{200}{\sqrt{3}+1}$
D. $200\sqrt{3}$
E. $400(\sqrt{3}-1)$

25. If the statement "All ducks in this pond are brown" is false, consider the following statements:

(1) No ducks in this pond are brown.
(2) At least one duck in this pond is not brown.
(3) There is at least one duck in this pond.

Which of the following must be true?

A. Only statement (1) must be true.
B. Only statement (2) must be true.
C. Only statements (1) and (3) must be true.
D. Only statements (2) and (3) must be true.
E. Statements (1), (2), and (3) must be all true.

26. Players A and B take turns removing either one or two bricks from a pile of n bricks, where n is a positive integer. Player A goes first, and the player who removes the last brick wins. For which values of n does Player A have a winning strategy (that is, a way to guarantee a win regardless of how B plays)?

A. Player A has a winning strategy if the number n of bricks is divisible by 3.

B. Player A has a winning strategy if the number n of bricks is even.

C. Player A has a winning strategy if the number n of bricks is not divisible by 3.

D. Player A always has a winning strategy, no matter what n is.

E. Player A has a winning strategy if the number n of bricks is odd.

27. Let $f(x) = \dfrac{2}{4^x + 2}$ for real numbers x. Evaluate $f\left(\dfrac{1}{3001}\right) + f\left(\dfrac{2}{3001}\right) + \cdots + f\left(\dfrac{3000}{3001}\right)$.

A. 1499

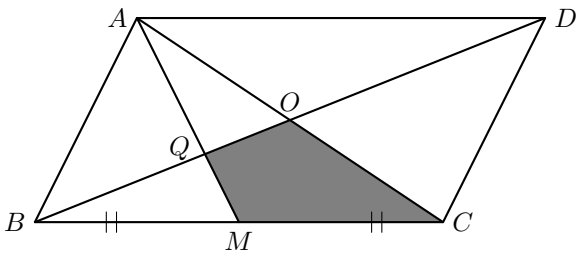
B. 1500

C. 1501

D. 3000

E. 3001

28. In parallelogram $ABCD$, the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O. Point M is the midpoint of $\overline{BC}$, and segment $\overline{AM}$ intersects diagonal $\overline{BD}$ at Q. The shaded region is quadrilateral $QOCM$. If the area of parallelogram $ABCD$ is 144, what is the area of the shaded region?



A. 18

B. 20

C. 24

D. 28

E. 30

29. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. How many subsets of A contain at least four consecutive integers?

A. 15

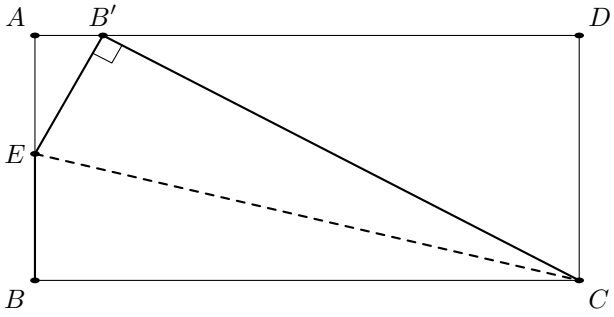
B. 18

C. 20

D. 24

E. 32

30. In rectangle $ABCD$, the rectangle is folded along line CE, where E is a point on side AB so that vertex B maps to a point B' on side AD. Given that $|\triangle AEB'| = 3$ and $|\triangle DB'C| = 27$. Find the area of rectangle $ABCD$.



A. $\frac{121}{2}$

B. $\frac{125}{2}$

C. $\frac{129}{2}$

D. $\frac{135}{2}$

E. $\frac{141}{2}$