State Math Contest 2026 Senior Level

Instructions:

- Calculators, cell phones and other computational devices are not permitted (you can only use pens, pencils and paper to work on your answers, and then mark your answers with a number two pencil on the answer sheet).
- Correct answers are worth 5 points each. Unanswered questions are worth 1 point each. Incorrect answers are worth 0 points each. *This means that it will not, on average, increase your score to guess answers randomly.*
- Fill in the answers on the answer sheet using a number two pencil.
- Time limit: 120 minutes.
- When you are finished, please give the exam and any scratch paper to the test administrator.
- Good luck!

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1. A circle has a radius of 10 cm. A chord is drawn 6 cm from the center. What is the length of the chord?

   A. 14 cm
   B. 16 cm
   C. 18 cm
   D. 15 cm
   E. 17 cm

2. A student is worried about their data usage. Consider one specific weekend, consisting of a Saturday followed by the next day, Sunday. On the Saturday of this weekend, the probability that the student exceeds their daily data limit is 40%, and on the Sunday of this weekend, the probability that the student exceeds their daily data limit is also 40%. Assume that exceeding the limit on Saturday and exceeding the limit on Sunday are independent events. What is the probability that the student exceeds their daily data limit at least once during this weekend?

   A. 0.16
   B. 0.36
   C. 0.40
   D. 0.54
   E. 0.64

3. What is the remainder when 6^{2026} is divided by 20?

   A. 6
   B. 10
   C. 12
   D. 16
   E. 18

4. If the statement “All ducks in this pond are brown” is false, consider the following statements:

(1) No ducks in this pond are brown.
(2) At least one duck in this pond is not brown.
(3) There is at least one duck in this pond.

Which of the following must be true?

A. Only statement (1) must be true.

B. Only statement (2) must be true.

C. Only statements (1) and (3) must be true.

D. Only statements (2) and (3) must be true.

E. Statements (1), (2), and (3) must be all true.

5. Consider the system of equations

$$ax + 4y = 8$$
$$3x + by = 6.$$

For how many pairs of positive integers (a, b) does this system have **no** solution?

A. 3

B. 4

C. 5

D. 6

E. 8

6. A number is considered “perfect” if the sum of its factors other than itself equals the number. For example, the factors of the number 6 are 1, 2, 3, and 6. When we add the factors other than 6, we get $1 + 2 + 3 = 6$. Which of the following numbers are perfect?

A. 17

B. 26

C. 128

D. 254

E. 496

7. Let A be the set of even non-negative integers which are less than or equal to 2026. Suppose a number is randomly chosen from A. What is the probability that this number is divisible by both 4 and 6?

   A. $\frac{168}{1013}$
   B. $\frac{169}{1013}$
   C. $\frac{28}{169}$
   D. $\frac{1}{6}$
   E. $\frac{85}{507}$

8. Solve the equation below.

$$\log_3(x+2) + \log_{\frac{1}{3}}(x-1) = \log_9(25)$$

   A. $\frac{7}{4}$
   B. $\frac{9}{8}$
   C. $\frac{1+\sqrt{93}}{2}$
   D. $\frac{1+\sqrt{29}}{2}$
   E. No Solution

9. Given $f(x) = (\log_2 x)^2 - 12\log_2 x + 32$. Find the domain of the function $g(x) = [f(x)]^{-2} + [f(x)]^2$.

   A. $(0,4) \cup (4,8) \cup (8,\infty)$
   B. $(0,4) \cup (4,256) \cup (256,\infty)$
   C. $(0,16) \cup (16,128) \cup (128,\infty)$
   D. $(0,16) \cup (16,256) \cup (256,\infty)$
   E. $(0,32) \cup (32,256) \cup (256,\infty)$

10. Below is a common proof humorously used to attempt to prove that $2 = 0$.

$$1 = 1 \tag{1}$$
$$-1 = -1 \tag{2}$$
$$\sqrt{-1} = \sqrt{-1} \tag{3}$$
$$\sqrt{\frac{1}{-1}} = \sqrt{\frac{-1}{1}} \tag{4}$$
$$\frac{\sqrt{1}}{\sqrt{-1}} = \frac{\sqrt{-1}}{\sqrt{1}} \tag{5}$$
$$\frac{1}{i} = \frac{i}{1} \tag{6}$$
$$i \cdot \frac{1}{i} = i \cdot i \tag{7}$$
$$1 = i^2 \tag{8}$$
$$1 = -1 \tag{9}$$
$$2 = 0 \tag{10}$$

$\square$

Of the 10 lines shown above, exactly one line follows from an invalid mathematical step. Which line is it?

A. Line 1

B. Line 3

C. Line 5

D. Line 7

E. Line 9

11. Find the derivative of

$$\ln\left(\frac{\sec^2 x \ln x}{x^2 + 5x + 4}\right)$$

A. $2\tan x + \dfrac{1}{x \ln x} - \dfrac{1}{x+1} - \dfrac{1}{x+4}$

B. $\dfrac{x^2 + 5x + 4}{\sec^2 x \ln x}$

C. $\dfrac{1}{2}\sin(2x) + \dfrac{1}{x} - \dfrac{2x}{x^2 + 5x + 4}$

D. $\dfrac{1}{2}\sin(2x) + \dfrac{1}{x \ln x} - \dfrac{1}{x+1} - \dfrac{1}{x+4}$

E. This function is not differentiable.

12. Let $F(x) = \int_{x^3}^{x^2} \frac{1}{1+t^2}\,dt$. Find $F'(x)$.

A. $\tan^{-1}(x^2) - \tan^{-1}(x^3)$

B. $\frac{1}{1+x^4} - \frac{1}{1+x^6}$

C. $\ln\left(\frac{1+x^6}{1+x^4}\right)$

D. $x^2\tan^{-1}\left(x^4\right) - x^3\tan^{-1}\left(x^6\right)$

E. $\frac{2x}{1+x^4} - \frac{3x^2}{1+x^6}$

13. If $\sin\theta - \cos\theta = \frac{1}{2}$, find the value of $\tan\theta + \cot\theta$.

A. $\frac{4}{3}$

B. $\frac{8}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

E. 2

14. Let $f(x)$ be a continuous function and satisfy $f(x+3) = -f(x)$ for all real x. On $[0,3]$, $f(x) = 2x^2 - 12x + 9$. Find $f(2025) + f(2026)$.

A. -10

B. -8

C. 8

D. -1

E. 1

15. Find the area of the region enclosed by the curves

$$y = x^2 - 2x + 1 \quad \text{and} \quad (x-2)^2 + y^2 = 1.$$

A. $\frac{\pi}{4} - \frac{1}{2}$

B. $\frac{\pi}{4} - \frac{2}{3}$

C. $\frac{\pi}{2} - \frac{1}{3}$

D. $\frac{\pi}{2} - \frac{2}{3}$

E. $\frac{\pi}{4} - \frac{1}{3}$

16. Let $a_1 = 1$ and $a_n = 4a_{n-1} + (-1)^n$ for $n \geq 2$. What is the ones digit of a_{2026}?

A. 1

B. 3

C. 5

D. 7

E. 9

17. A pizza shop offers 7 different toppings: pepperoni, ham, mushrooms, onions, olives, peppers, and sausage. You may choose any subset of these toppings (including none), and you may choose each topping at most once. However, you may not choose two toppings that are consecutive in the list (the list is in the order given above). How many different topping selections are possible?

A. 21

B. 24

C. 34

D. 44

E. 55

18. Given $\triangle ABC$ with $AB = 3$, $AC = 4$,and $\angle A = 90°$, let G be the centroid (center of mass) of the triangle. Let H be the point on $\overline{AB}$ such that $\overline{GH} \perp \overline{AB}$. Find the length of $\overline{GH}$.

   A. $\dfrac{2}{3}$
   B. $\dfrac{4}{3}$
   C. 2
   D. $\dfrac{8}{3}$
   E. $\dfrac{16}{3}$

19. Find $\lim_{h \to 0} \dfrac{(1+h)^{2+2h} - 1}{h}$.

   A. 2
   B. 0
   C. $e^2 - 1$
   D. $2\sqrt[3]{e}$
   E. ∞

20. Consider two parabolas in the coordinate plane:
   Parabola 1: $y = x^2$
   Parabola 2: $y = -x^2 + 4x - 10$
   These two parabolas share two common tangent lines. Find the equation of the tangent line that has a positive slope.

   A. $y = 6x - 9$
   B. $y = 2x - 1$
   C. $y = 5x - \dfrac{25}{4}$
   D. $y = 4x - 4$
   E. $y = 8x - 16$

21. Let f be a real-valued function such that $f''(x) = 1$ on $[0, 10]$. Let

$$s = 2[f(1) + f(3) + f(5) + f(7) + f(9)].$$

Which of the following must be true?

A. $s < \int_0^{10} f(x)dx.$

B. $s > \int_0^{10} f(x)dx.$

C. $s = \int_0^{10} f(x)dx.$

D. $s > \int_0^{10} f(x)dx$ if and only if f is increasing

E. None of the above must be true.

22. A robot must travel from point $(0, 0)$ to point $(5, 5)$ on a coordinate plane. At each step, the robot may move either one unit up or one unit to the right. Due to a "glitch," the robot is not allowed to pass through the point $(2, 2)$. How many distinct paths can the robot take?

A. 120

B. 132

C. 192

D. 234

E. 252

23. Let z be a complex number such that

$$|z - 2| = |z - (4 + 2i)|.$$

What is the minimum possible value of $|z|$?

A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. 4

E. $4\sqrt{2}$

24. Fifteen people are seated at a round table. A committee of 3 people is chosen at random from the 15 people. Find the probability that at least two of the people chosen were sitting next to each other.

A. $\dfrac{4}{15}$

B. $\dfrac{3}{5}$

C. $\dfrac{36}{91}$

D. $\dfrac{1}{2}$

E. $\dfrac{43}{120}$

25. Find the sum $a+b$ for integers a and b such that $\dfrac{2}{a}+\dfrac{1}{b}-\dfrac{2}{ab^2}=\dfrac{1}{3}$.

A. -11

B. -7

C. 1

D. 11

E. 13

26. Rewrite $\displaystyle\sum_{k=0}^{n}\frac{1}{(n-k)!(n+k)!}$ in a closed form.

A. $\dfrac{2^{2n-1}}{(2n)!}+\dfrac{1}{(n!)^2}$

B. $\dfrac{2^{2n}}{(2n)!}+\dfrac{1}{2(n!)^2}$

C. $\dfrac{2^{2n-1}}{(2n)!}+\dfrac{1}{2(n!)^2}$

D. $\dfrac{2^{2n+1}}{(2n)!}+\dfrac{1}{(n!)^2}$

E. $\dfrac{2^{2n+1}}{(2n)!}+\dfrac{1}{2(n!)^2}$

27. For the real numbers of x, y, z, w such that $0 \leq x, y, z, w \leq 1$, find the maximum of $x(1-y) + 2y(1-z) + 3z(1-w) + 4w(1-x)$.

A. 4

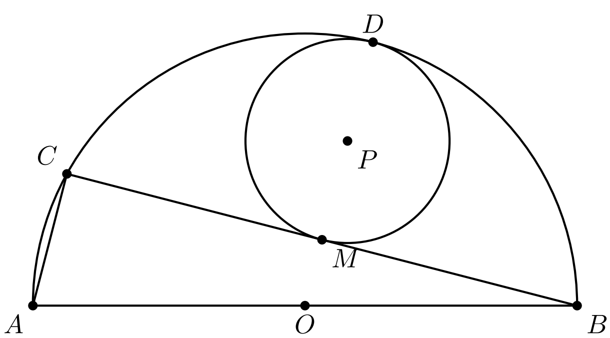
B. 5.5

C. 6

D. 7

E. 7.5

28. Let $\overline{AB}$ be the diameter of a semicircle with center $O = (0, 0)$ and let C be a point on the semicircle. Let M be the midpoint of $\overline{BC}$. A circle is tangent to $\overline{BC}$ at M and internally tangent to the semicircle at point D. Let P be the center of that circle. If $AB = 40$ and $AC = 10$, find the x-coordinate of P.



A. $\frac{5}{4}$

B. $\frac{35}{8}$

C. $\frac{15}{4}$

D. $\frac{25}{8}$

E. $\frac{25}{4}$

29. Point P is located inside triangle $\triangle ABC$ so that the angles $\angle PAB$, $\angle PBC$, and $\angle PCA$ are all congruent. The sides of the triangle have lengths $AB = 15$, $BC = 16$, $CA = 17$. Find $\tan \angle PAB$. $\Big[$Hint: Heron's formula: If a triangle have side lengths a, b, and c, then the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$. $\Big]$

A. $\dfrac{96\sqrt{21}}{15^2+16^2+17^2}$

B. $\dfrac{24\sqrt{21}}{15^2+16^2+17^2}$

C. $\dfrac{48\sqrt{21}}{15^2+16^2+17^2}$

D. $\dfrac{20\sqrt{21}}{15^2+16^2+17^2}$

E. $\dfrac{144\sqrt{21}}{15^2+16^2+17^2}$

30. Alice and Bob each secretly choose an integer from 1 to 30 (inclusive). They know the range and they know that each other reasons perfectly.

They take turns asking questions. When asked a yes/no question, a person answers:

- **Yes** if the statement must be true given what they know,
- **No** if the statement must be false given what they know,
- **I don't know** if either answer is possible.

The conversation is:

1. Alice: Is your number double mine?
2. Bob: I don't know. Is your number double mine?
3. Alice: I don't know. Is your number half mine?
4. Bob: I don't know. Is your number half mine?
5. Alice: I don't know.
6. Bob: Now, I know your number.

What is Alice's number?

A. 4

B. 8

C. 12

D. 16

E. 20