The Effect of Different Instructional Methods and Beliefs Towards Mental Computations

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Abstract

The purpose of this research are threefold: 1) to compare the outcomes of preservice teachers' mental computation performance of whole numbers, integers, and rational numbers, using the three different instructional approaches (i.e., direct teaching, open approach, and control group), 2) to identify which operations showed a marked difference on the Mental Computation Test (MCT) performance for each group, and 3) to measure the changes in positivity or negativity of belief towards learning written and mental computations after an intervention. PTs in the K-8 Teacher Education Program participated in this study. A mixed research method is used for this study. Specifically, a quasi-experimental design is employed using a pre-and post-MCT that consists of 69 items in relation to whole numbers, integers, and rational numbers (i.e., fractions, decimals, and percents). A one-way analysis of covariance is used to reveal if there is a statistically significant difference in post-MCT scores. In addition, the Mathematics Attitudes Survey (MAS) is designed to analyze preservice teachers' positivity or negativity of belief regarding their mental computation and written computation learning. This study aims to contribute to the existing body of research that provides useful insights for mathematics educators on how to effectively apply instructional approaches to promote diverse students' mathematics knowledge of mental computations and to provide useful information as a measure of current preservice teachers' mental computation ability and beliefs on mental computation.

Keywords: Mental computations, direct teaching, open approach, preservice teachers, attitudes towards learning mental computations

Introduction

Today, more studies pay attention to the success using mental computation and try to determine its influences on students' achievement in and out of school (Yang & Huang, 2014); however, the main focus of mathematical computation in the primary school has been placed on written pencil and paper algorithms. Since many classroom teachers have been educated in ways that focus on the rote memorization of basic facts, and the development of procedures for completion of traditional written algorithms, their teaching strategies are accordingly influenced by their previous learning experiences. Although these teachers can see benefits for using mental computation strategies in their classrooms, their lack of related knowledge has led to a lack of confidence and teaching skills (Hartnett, 2007). It is even more doubtful how effectively the preservice teachers use the strategies they have developed. To succeed in learning and in teaching mental computation to students, it is important for preservice teachers to be prepared to teach effectively prior to classroom teaching.

There are two growing different instructions: direct teaching and developing students' own strategies (Hartnett, 2007; Varao & Farran, 2007). Even though the direct teaching instruction originally came from a behavioristic approach, many researchers agree that the direct teaching should be involved in students' conceptual understanding along with their procedural skills (Reys, Reys, Nohda, & Emori, 1995; McIntosh, Nohda, Reys, and Reys, 1995). The second approach, developing students' own strategies, comes from a constructivist view (Becker & Epstein, 2007; Hartnett, 2007; Becker & Shimada, 1997). Teachers can create this environment by encouraging students to solve problems in a variety of ways (Becker & Epstein, 2007; Hartnett, 2007). Using the *open approach* with constructivist instructional benefits to deepen students' mathematics understanding and content knowledge. *Open approach* problems are those for which there are multiple correct answers or ways of solving the problems. The results of the *open approach* showed that students have an opportunity to be more actively involved in lessons, to deepen their mathematics learning, and to enjoy their experiences in problem-solving (Becker & Epstein, 2007).

With respect to teaching practice, Jong and Hodges (2015) investigated the attitudes towards mathematics among preservice elementary teachers in relation to their experiences with K-12 learners of mathematics and experiences in a teacher education program. The result showed that developing positive attitudes was an important aspect of teacher education as attitudes influence the instructional practices preservice teachers use with students.

Thus, this study is closely connected to the Sustainable Development Goal 4 (SDG 4), Target 4.1, *by 2030, ensure that all girls and boys complete free, equitable and quality primary and secondary education leading to relevant and effective learning outcomes*. This means that mental computation skills can be used as an essential key concept for developing both preservice teachers and their future students' mathematics proficiency and understanding.

Purpose and Research Questions

The purpose of the current study is twofold. The first part of the study seeks to compare the outcomes of preservice teachers' performance in mental computation of whole numbers, integers, and rational numbers using the three different instructional approaches (i.e., direct teaching, open approach, and control group) and to identify which operations showed a marked difference on the Mental Computation Test (MCT) performance for each group. The second part measures the changes in positivity or negativity of belief towards learning written and mental computations after an intervention using the two approaches (i.e., the direct teaching approach and the open approach). The specific research questions that guided this study were as follows:

1. Are there any significant differences in mental computation performance between the experimental groups (i.e., *Direct Teaching* and *Open Approach*) and the control group before and after instruction?

2. Which mathematical operations showed a marked difference in improvement of pre- and post-MCT performance among the three instructional approaches?

3. How do the preservice teachers' mathematical beliefs towards written and mental computation change after the intervention?

Methodology

This study used a quasi-experimental, non-equivalent control group design. The convenient sampling design was used.

Participants

The population is a group of preservice teachers enrolled in a K-8 teacher education program at a mid-sized, four-year, state university in the USA. The convenient sampling design was used for this study. The sample size was 50 preservice teachers before the intervention and 40 after the intervention. Ten students were dropped out during the semester. Three classes were used for this study: two classes experimental groups enrolled in Course A, and one control group. Course A is the first required class, which provides an overview of a real number system, operations on whole numbers, arithmetical skills using mathematical activities and mathematical thinking, and problems solving skills. In terms of mathematics attitudinal surveys (MAS), 27 of the 50 participants received the intervention. 20 participants from the control group did not take part in the post-MAS because no intervention was provided for them. 3 participants were dropped during the intervention.

Test Instrument

The researcher designed and constructed the Mental Computation Test (MCT) to determine whether there were significant changes in preservice teachers' mental computation performance. A pilot study was conducted in the previous semester. After the piloting of the instrument, we identified weaknesses in the structure of the MCT and revised the test. The MCT included 69 problems in relation to whole numbers, integers, and rational numbers (i.e. fractions, decimals, and percents). The MCT was administered using PowerPoint slides. The questions were displayed one at a time on PPT slides for about 30 seconds. Each item of the MCT was assigned one point for a correct answer and no point for an incorrect answer or no response. The reliability of the MCT items was calculated using Cronbach's alpha coefficient ($\alpha = 0.96$). Three mathematics educators ascertained the content validity of the test.

The researcher designed the Mathematics Attitudes Survey (MAS). The content and face validity of MAS was reviewed by three mathematics educators and pilot tested. The necessary items for each operation were modified, deleted or added. Descriptive statistics were utilized to analyze the results. The Likert-type scale consisted of 30 items and clustered by the following two categories: preservice teachers' perception about mental and written computation (16) and PTS' perception of instruction between written and mental computation (14). Two types of statements were included in a parallel way – each statement was accompanied by a parallel statement. Cronbach's alpha test was calculated to check the internal consistency reliability (30 items: $\alpha = 0.83$).

Procedure for Intervention

For the first session, the pre-MCT and pre-MAS were administrated. The researcher carried out the intervention sessions (i.e., 11 sessions over 8 weeks) for the whole numbers, integers, and rational numbers during the semester. As for the first session, the participants completed pre-MAS. After that, in the two experimental groups, the MC strategies that were more focused on conceptual understanding were respectively implemented using the following two instructional methods: the direct teaching and the open approach. As for the direct teaching, the researcher introduced and demonstrated several MC strategies in the lessons using more flexible deductive strategies such as compensation (e.g., $20 \times 199 = 20 \times (200 – 1) = 20 \times 200 – 20 \times 1 = 4{,}000 – 20 = 3{,}980$), dividing using factors (e.g., $70 \div 14 = (70 \div 7) \div 2 = 5$), number facts (e.g., 9% × 450 = 10% of 450 – 1% of 450 = 45 – 4.5 = 40.5), and so on. For the open approach group, an open-ended problem was first presented progressing from easy to more complicated. The participants were asked to find solutions in many ways using their own natural thinking abilities. Then, selected PTs shared or explained how they solved their problems on the board for the whole class to see. Then we discussed what solutions were the most appropriate for a given problem. No intervention is provided for the control group. In the last session, preservice teachers were given both post-MCT and post-MAS.

Analyzing Data

To examine differences in mental computation performance between the three groups before and after instruction, and to examine a marked difference for each operation with different groups, a one-way ANOVA was conducted to determine whether there was a significant difference in the mean pre-MCT scores. An Analysis of Covariance (ANCOVA) was conducted on post-MCT performance, with the type of instruction (i.e. direct teaching, open approach, and Control) and pre-MCT scores as covariates to control for pre-MCT score differences among the groups. Descriptive and inferential statistics were used to analyze the results of the Mathematics Attitude Survey (MAS).

Findings

Differences Between Pre-and Post-Mental Computation Test Performance

With respect to inferential statistics, a one-way analysis of covariance (ANCOVA) was conducted for this study. Levene's test and normality checks were carried out and the assumptions of normality and homogeneity of variance were met. The result of the Levene's test showed the variances are not unequal, $F(2, 38) = 0.207$, $p = 0.814$.

Table 1

Overall descriptive statistics comparison between experimental groups and the control group on the pre-and post-mental computation test.

TEST	Direct Teaching			Open Approach			Control		
	N	M	SD	N	M	SD	N	M	SD
Pre-MCT	14	27.86	14.57	16	32.38	16.05	20	35.80	14.21
Post-MCT	11	40.08	14.00	15	44.60	18.82	14	34.21	14.44
Improvement	11	9.50	5.58	15	11.87	6.85	14	-3.27	7.62

Table 2 summarizes the one-way ANCOVA result for the post-MCT by instructional condition and pre-MCT scores. There was a statistically significant difference F (2, 37) = 17.52, p < .05, at the .05 level, in post-MCT scores between the different instructional groups, when adjusted for pre-MCT scores. Accordingly, both the observed and adjusted means showed that although the open approach group performed better than the direct teaching group, the means of both groups have increased with a similar amount (i.e., M= 9.5 vs M= 11.87) since the PTs in the open approach group started off higher. The PTs in the control group performed the worst.

Table 2

ANCOVA results and descriptive statistics for post-MCT.

Type of Groups	Mathematics Scores			
	Observed Mean	Adjusted Mean	SD	n
Direct Teaching	40.08	42.79	14.00	12
Open Approach	44.60	45.29	18.81	15
Control	34.21	31.16	13.50	14

Source	SS	df	MS	F
Pre-MCT	7825.58	1	7825.58	174.50*
Instruction	1570.92	2	785.46	17.52*
Error	1659.29	37	44.85	

Note. R^2 = .84, Adj. R^2 = .83, adjustments based on Pre-MCT mean = 39.75. Homogeneity of regression tested and not significant: F = 1.40, p>.05. Pre-MCT regression coefficient = 0.88*. * p < .05

As indicated by Table 3, multiple comparisons showed that there was a significant difference between the direct teaching and control groups (p < 0.05) and the open approach and control (p < 0.05) groups. However, these two groups did not significantly differ on their post-MCT scores.

Table 3

Multiple comparisons and mean differences in post-MCT scores by instruction type controlling for pre-MCT scores.

Comparison	Mean Difference	Standard Error of Difference	Bonferroni Adjusted (95% CI)
Direct teaching vs. Open approach	-2.49	2.60	-9.01, 4.02
Direct teaching vs. control	11.64*	2.67	4.94, 18.33
Open approach vs. control	14.13*	2.51	7.85, 20.41

Note. Comparisons based upon ANCOVA adjusted means controlling for Pre-MCT mean scores of 39.75. * p <.05, where p-values are adjusted using the Bonferroni method.

Operations that Showed a Marked Difference

Table 4 summarizes the comparison of three different groups with different instructions for each operation between pre-and post-MCT scores. There were significant differences between pre- and post-MCT performance among the three groups in solving multiplication, fraction, and decimal operations. More specifically, the one-way ANCOVA for mental multiplication performance was significant, F (2, 23) = 8.48, p = .002, η_p^2 = 0.43. The effect size (η_p^2) = 0.43 is quite large. The pairwise comparisons indicated that there was a significant difference between the open approach and control groups (p = 0.006). Additionally, the ANCOVA for decimals, F (2, 26) = 5.88, p = .008, η_p^2 = 0.31 showed a significant result and large effect size. The pairwise comparisons showed that there were significant differences between the direct teaching and control groups (p = 0.026) and between the open approach and control groups (p = 0.023). A significant difference between the experimental groups was not present for the pairwise comparisons.

Table 4

Comparison of type of instruction for the operation between pre-and post-MCT scores.

Operation	# of item	Test (%)	Direct Teaching		Open Approach		Control		*F*	η_p^2	*p*
			M	SD	M	SD	M	SD			

Subtractions	5	Pre	61.60	29.59	65.20	20.07	76.00	12.94	2.57	.32	.235
		Post	80.00	19.22	73.40	8.17	65.60	26.71			
Multiplications	9	Pre	15.00	19.40	18.78	21.27	23.89	21.33	8.48*	.43	.002
		Post	38.89	25.97	39.33	17.44	22.11	13.68			
Integers	2	Pre	36.00	9.90	56.50	9.19	42.50	10.61	.80	.44	.556
		Post	45.50	17.68	63.00	14.14	46.50	14.85			
Fractions	8	Pre	28.63	14.43	45.50	20.07	41.88	15.80	4.66*	.32	.022
		Post	46.75	17.38	65.75	14.00	39.25	19.03			
Decimals	10	Pre	48.00	23.36	56.30	25.11	57.00	19.18	5.88*	.31	.008
		Post	67.50	18.63	76.80	13.05	52.90	22.42			
Percents	6	Pre	26.00	24.62	28.33	32.10	35.00	28.28	.98	.40	.401
		Post	32.00	27.66	45.50	20.71	32.00	24.24			

Note. η_p^2 = Partial Eta-Squared. * $p < .05$

Mean Changes in Attitudes towards Mental and Written Computations

Changes in the mean score of attitudes can be examined visually as indicated by Table 5. As for the pre-MAS, the minimum score was 71, with a maximum score of 129, out of a possible 150, indicating a rather wide range of attitudes at the beginning of the course. The mean pre-MAS score was 103.24 (SD =10.11). On the post-MAS, the minimum score was 70 and the maximum score was 115, indicating a narrower range of attitudes at the end of the course. The mean score for the post-MAS was 104.85 (SD = 9.29). Although the total standard deviation between mental and computation showed not much difference, the difference towards written computation between pre-and post-MAS showed a wide range of scores on attitudes. It could be interpreted that preservice teachers’ attitudes towards written computations were changed.

Table 5

Descriptive Data for Differences towards Mental and Written Computations.

		Minimum Raw Score	Maximum Raw Score	Mean*	Standard* Deviation	N
Pre-MAS	Total	71	129	103.24	10.11	30
	Mental	32	71	54.78	6.84	
	Written	32	60	48.46	5.44	
Post-MAS	Total	70	115	104.85	9.29	27
	Mental	33	74	58.70	8.70	
	Written	20	56	46.15	9.17	

Note. MAS: Mathematics Attitudinal Survey, * Rounded to nearest hundredth

Tables 6 and 7 show the mean score changes of the two experimental groups. The positive changes are mostly related to written computation. The greatest positive mean change between pre and post was 0.44: “I believe WC is more useful in real life situations.” This was followed with: “I have spent more time in school doing written computation than mental computation.” (Mean Changes (MC) = 0.40); and “I am confident with learning and teaching written computation (MC = 0.39). The negative changes were mostly connected to the mental computation. There were two negative changes that were greater than 0.25. First, “I believe MC is more useful in real life situations” decreased with a mean change of 0.34 between the pre and post surveys. Second, with a negative mean change of 0.33 was: “Mental computation should be taught during the school years.”

Table 6

Mean Changes in belief towards Mental and Written Computations.

	Pre-MAS			Post-MAS			Mean Change
	N	Mean	SD	N	Mean	SD	
1. I have learned WC strategies during my school years.	30	3.93	0.74	27	3.93	0.92	-0.01
2. I have learned MC strategies during my school years.	30	3.50	0.86	27	3.41	1.05	-0.09
3. I have spent more time in school doing WC than MC.	30	3.90	0.96	27	4.30	0.78	0.40
4. I have spent more time in school doing MC than WC.	30	2.13	0.90	27	2.15	0.82	0.31
5. I feel comfortable and safe when using WC.	30	3.83	1.12	27	4.15	0.86	0.31

6. I feel comfortable and safe when using MC.	30	2.77	0.97	27	2.74	0.98	-0.03
7. I am confident with learning and teaching WC.	30	3.87	0.94	27	4.26	0.71	0.39
8. I am confident with learning and teaching MC.	30	2.87	1.04	27	3.04	1.13	0.17
*9. I have used WC more than MC.	30	4.03	0.89	27	4.33	0.78	0.30
10. I have used MC more than WC.	30	2.07	0.78	27	2.11	0.97	0.04
11. I believe WC is more useful in real life situations.	30	2.93	0.91	27	3.37	1.11	0.44
12. I believe MC is more useful in real life situations.	30	3.57	0.82	27	3.22	0.89	-0.34
13. WC should be taught during the school years.	30	4.10	0.71	27	4.26	0.71	0.16
14. MC should be taught during the school years.	30	4.03	0.72	27	3.70	1.10	-0.33
15. WC is easy to learn and solves problem quickly.	30	3.73	0.87	27	3.93	0.73	0.19
16. MC is easy to learn and solves problems quickly.	30	3.30	0.79	27	3.15	1.20	-0.15

Note. WC: Written Computation; MC: Mental Computation

There were also negative and positive changes between the pre- and post- surveys on instruction in mental and written computations as shown in Table 7. The positive changes were mostly found in written computation: “Written computation should be introduced first when teaching mathematics” (MC = 0.34). There were also positive increases indicating that “I think I will use WC more when I teach students (MC = 0.32)”. Also, survey results showed that “students who are highly skilled in WC develop problem-solving skills (MC = 0.25).” The surveys’ negative changes mostly related to the MC. The greatest negative change in means between pre and post was - 0.21: “Students can be successful mathematics learners by teaching only MC.” This was followed with: “I think I will use MC more when I teach students” (MC = -0.11) and “Mental computation should be introduced first when teaching mathematics” (MC = -0.10).

Table 7

Mean Changes in Instruction towards Mental and Written Computation.

	Pre-MAS			Post-MAS			Mean Change
	N	Mean	SD	N	Mean	SD	

17. I think I will use WC more when I teach students.	30	3.53	0.82	27	3.85	0.91	0.32
18. I think I will use MC more when I teach students.	30	2.97	0.85	27	2.85	1.06	-0.11
*19. Students can be successful mathematics learners by teaching only WC.	30	2.48	0.78	27	2.70	1.10	0.22
20. Students can be successful mathematics learners by teaching only MC.	30	2.43	0.73	27	2.22	0.75	-0.21
21. WC should be introduced first when teaching mathematics.	30	3.73	0.91	27	4.07	0.87	0.34
22. MC should be introduced first when teaching mathematics.	30	2.80	1.00	27	2.70	1.17	-0.10
*23. Teaching WC can build students' mathematical procedural knowledge and understanding.	30	4.00	0.53	27	4.22	0.42	0.22
24. Teaching MC can build students' mathematics procedural knowledge and understanding.	30	3.70	0.84	27	3.70	0.91	0.00
25. WC should be taught to learn advanced mathematics.	30	3.73	0.83	27	3.96	0.90	0.23
26. MC should be taught to learn advanced mathematics.	30	3.40	0.89	27	3.33	1.00	-0.07
27. Students can develop their natural thinking ability through learning WC.	30	3.73	0.74	27	3.81	0.74	0.08
28. Students can develop their natural thinking ability through learning MC.	30	3.83	0.59	27	3.78	0.89	-0.06
29. Students who are highly skilled in WC develop problem solving skills.	30	3.60	0.77	27	3.85	0.77	0.25
30. Students who are highly skilled in MC develop problem solving skills.	30	3.60	0.77	27	3.74	0.66	0.14

Note. WC: Written Computation; MC: Mental Computation

Research Limitations and Implications

The major findings of this study as explicit partial answers to the three research questions are briefly summarized as follows. When examining preservice teachers' differences in mental computation performance between the experimental groups and the control group, experimental groups (i.e., open approach and direct teaching) performed better than the control group. The level of improvement in the post-MCT scores of the direct and open approach groups was not significantly different. When comparing the performance of experimental groups, the open approach group performed better than the direct teaching group. However, this study found that direct teaching instruction involving students' conceptual understanding may be equally effective in improving preservice teachers' performance of MCT as the open approach.

There were significant differences between pre-and post-MCT performance among the three groups in computing whole number multiplication, operations with fractions, and decimal operations (See Table 4). Specifically, studies found that learning fractions and decimals are difficult for students to master (Bailey et al., 2012; Hiebert and Wearne, 1985; Lortie-Forgues el al., 2015; Siegler et al., 2011). It is important results because mental computation using direct teaching or open approach can build students' ability to compute fraction and decimal operations.

As for the mean changes in preservice teachers' beliefs towards mental and written computation, there were negative and positive changes. The positive changes mostly related to written computation, while the negative changes were mostly connected to mental computation. Similar results of the pre-and post-MAS on instruction regarding mental and written computation were found. The positive changes were mostly related to written computation, while negative changes were mostly connected to mental computation. Interestingly, this study revealed that before the intervention, participants were well-aware of the importance of learning mental computation and they believe that both written and mental computations should be taught during the school years; however, after intervention, the levels of beliefs towards learning mental computation was decreased and participants put more emphasis on using written computation. This may be due to their lack of mental computation skills, so this leads to relying more heavily on written computation because it is still more familiar to them.

Five implications for future study are as follows. First, more empirical studies comparing the effect of direct teaching and open approach instructions are needed. Several studies (Becker and Epstein, 2007; Kwon et al., 2006; Lin et al., 2013) found that open approach instruction results in significant learning gains in comparison to traditional instruction; however, for this study, the results of the open approach group performance did not show any significant increases in MCT performance compared to the direct teaching group. This is an interesting finding because during the intervention, the researcher observed PTs' active participation and a variety of solution methods that cultivate students' flexibility and creativity. A possible explanation for these results may be the lack of adequate time and test anxiety.

Second, Kirschner et al. (2006) assert that direct instruction is needed for low-achieving students and *unguided instruction* (i.e., open approach) is effective for more able learners. The findings of this study would be different if the researchers conducted this study based on students' different levels of achievements. Further studies, which take these variables into account, need to be undertaken.

Third, it will be necessary for researchers to conduct power analyses to determine minimum sample sizes for studies. Three operations that indicated non-significant results (See Table 4) showed larger effect size (i.e., subtraction ($\eta p^2 = 0.32$), integers ($\eta p^2 = 0.44$), and percents ($\eta p2 = 0.40$)) although non-significant results were shown. Thus, it could be interpreted that non-significant results may be due to lack of power rather than lack of effect.

Fourth, the findings of this study confirmed that preservice teachers were more likely to use written computation than mental computation when solving mathematics problems and revealed that they did not predict their success of future mental computation teaching due to a lack of mental computation knowledge and confidence. To have students consistently practice mental computation in their K-12 mathematics classrooms or to use mental computation skills in their real-life situation, it is imperative for mathematics educators and other mathematics stakeholders to include mental computation in the U.S. K-12 mathematics curriculum, specifically, the mathematical content standards.

Last, mental computation is currently used in every culture by students (e.g., Reys et al., 1995, Yang and Huang, 2014), but few studies comparing preservice teachers' attitudes and beliefs on mental computation have been done across cultures. Therefore, a cross-cultural study of mental computation knowledge employed by the preservice teachers would be a productive implication for further study.

Limitations and Recommendations

The current research has several limitations. First, this study used a quasi-experimental, nonequivalent control group design. It is difficult to generalize the findings of the study because of the small sample size and a convenience sampling. Second, it is difficult to control for threats to internal and external validity of the study. Only 50 PTs in the pre-MCT and 40 PTs in the post-MCT participated in this study. Mortality or attrition is one of the potential threats to the internal validity of the study. Treatment diffusion (i.e. different treatment groups communicate with and learn from each other) is one of the potential threats to the external validity of the study. Third, only eleven intervention sessions were provided including test sessions and those were not enough time to practice mental computation strategies using multiple solution methods. Lastly, the validity of instruments such as pre-and post- MAS may not represent the actual construct because the researcher created the instrument. The evidence of instrumental validity was not thoroughly examined except for the content validity examined by three mathematics experts.

Originality/Value of the Paper

There is a lack of research comparing how these two alternative instructional approaches (i.e., direct teaching vs. open approach) impact the ability of preservice teachers' mental computation learning of whole numbers, integers, and rational number. Thus, this study aims to contribute to the existing body of research that provides useful insights for mathematics educators on how to effectively apply instructional approaches to them to be able to do mental computations and to identity students' in-depth insights on attitudes towards mental and written computations. Also, preservice teachers' mathematics attitudes and beliefs towards written and mental computations have not been adequately studied and reported. Thus, this study may help mathematics educators in this regard.

Conclusion

To succeed in learning and in teaching mental computation to students, it is important for preservice teachers to be prepared to teach effectively prior to classroom teaching. Teachers' beliefs in mathematics teaching and learning may play a vital role in students' understanding of mental computation**.** The findings of this research will contribute to the research base that is related to preservice teachers' knowledge of mental computation. In other words, if the mental computation is an ability student should develop and improve, it should be analyzed what classroom instruction works best to encourage preservice teachers' mental computation ability. Viewing mental computation as higher-order thinking requires preservice teachers to learn their instructional techniques. Also, mathematics educators in preservice, inservice, and professional developmental programs may apply mental computations that work best for different learners. Finally, the findings of this study may provide useful information to mathematics teacher educators and educational policy makers, which may enhance existing teacher preparation programs and preservice teachers' attitudes toward mathematics teaching and learning. Therefore, this research help math educators ensure that *by 2030, all girls and boys complete free, equitable, and quality primary and secondary education leading to relevant and effective learning outcomes*, as indicated in Sustainable Development Goal 4 (SDG 4), Target 4.1.

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