Addressing Concerns About Instant Runoff Voting

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Executive Summary

This report discusses concerns about instant runoff voting (IRV) that have arisen about how IRV behaves in both theory and practice. To address these concerns, we also describe some of the mathematics of voting.

- We describe briefly several fairness criteria in the theory of voting including
  - ◊ The Monotonicity Criterion
  - ◊ The Condorcet Winner and Condorcet Loser Criteria
  - ◊ The Independence of Irrelevant Alternatives Criterion (i.e., avoiding spoilers)
- We compare various voting methods including plurality, instant runoff voting (IRV), ranked pairs (RP), and score voting (SV) on which fairness criteria they satisfy.
- We address several general concerns about IRV as well as specific concerns brought up in a recent technical report by Jiri Navratil and Warren Smith (Navratil & Smith, 2022). We also compare these concerns to how well plurality fares on the same topic.
- We conclude that
  - ◊ IRV ballot error rates are generally small and are unlikely to affect the outcome of an election.
  - ◊ IRV is immune to the main type of spoiler that plurality is susceptible to, but is susceptible to other kinds of spoiler candidates.
  - ◊ IRV fails the Condorcet Winner Criterion but satisfies the Condorcet Loser Criterion. Plurality fails both Condorcet criteria.
  - ◊ IRV fails the Monotonicity Criterion, while plurality satisfies it.
  - ◊ Both IRV and plurality cause strategic voting, as do all voting methods. IRV appears superior to plurality at minimizing strategic voting.
  - ◊ IRV can result in different outcomes than plurality.
  - ◊ IRV does not throw out ballots but rather uses them until there is no longer any relevant information left on them.
  - ◊ IRV elects a majority winner among those voters that indicated that they wanted to have a say between the candidates remaining in the final round. No other majority makes sense to require.
    - * Voters can ensure that IRV always elects a majority winner among all votes cast if they all fill out a complete ranking.
  - ◊ IRV fails the Participation Criterion, as do most voting methods. The Participation Criterion will not affect how voters choose to vote because to use it strategically requires information that is not available until after the election. Thus, IRV's failure of this criterion is not concerning.
  - ◊ IRV, like all voting methods including plurality, falls victim to some voting paradoxes. Balancing the paradoxes to which an election system is susceptible with that system's potential benefits is the key question in determining which system to use.
- In our experience, IRV is generally considered mathematically superior to plurality, which is widely considered to be mathematically one of the worst ways to vote. Continuing to explore IRV also has the advantage that it continues the conversation of improving our election method.
- In the Appendix, we provide more details on the mathematics of voting.
- Game theory, the branch of mathematics that studies how "players" make decisions, is applied to voting. By so doing, we can analyze the effects of various voting methods.
  - ◊ The purpose of voting is to accurately determine the collective opinion of the people about which candidate is preferred.
  - ◊ The goal of an election method is to accomplish that purpose while incentivizing honest voting and civil elections.
  - ◊ We should judge the utility of a voting method on how well it satisfies the purpose of voting and achieves the goals of an election method.
- We describe in more detail various fairness criteria and two important mathematical theorems about whether voting methods can satisfy all of them.
- Glossary of acronyms used
  - ◊ RCV – Ranked Choice Voting
  - ◊ IRV – Instant Runoff Voting
  - ◊ RP – Ranked Pairs
  - ◊ SV – Score Voting

Introduction

In any democracy where rule by the people is important, how to ascertain the voice of the people is critical, regardless of whether the people's voice is electing representatives or directly determining what the group should do. The United States of America is one of the oldest modern democracies, though its government is more precisely described as a constitutional federal democratic republic (108th Congress of the United State of America, 2003). Since the United States is a republic, the main decisions made directly by the people are electing representatives, but the people often directly make decisions through ballot initiatives. In either case, the voice of the people is identified through voting. Since electing representatives is the main use of voting in Utah and the United States, we will restrict our attention in this report to that purpose, but the principles discussed here could be applied to other types of elections.

At first glance, voting may seem to be a simple matter conceptually, and indeed it is if there are only two options to be made. Each voter selects which of the two options they prefer, and whichever option is selected by the majority of the voters is the option considered to be collectively selected by the group. But the situation becomes more complicated if there are more than two potential choices. It can also become more complicated for other reasons, such as when a federation of states attempts to elect a chief executive or when a state tries to elect several representatives according to population and must determine which groups in the state elect which representatives. However, we will restrict ourselves here to the problem of how to elect a representative when there are more than two potential choices.

Two choices of voting methods that seem to have garnered the most public interest lately are single-choice voting (also known as plurality voting) and instant runoff voting (IRV), though some more popularly refer to IRV as ranked choice voting (RCV). A debate on which of these two methods is superior is currently underway in the state of Utah.

Since January 2019, Utah has participated in an RCV pilot program (Utah Code 20A-4-6, 2018) that was passed with great consensus, with only three state representatives voting against it and no state senators voting against it (Utah State Legislature 2018 General Session, 2018). In the years that the pilot has run, however, many citizens and several state legislators have raised concerns about the effects of RCV and how it compares to the traditional single-vote election method called plurality. Some of the concerns are based on mathematical issues that may exist within plurality voting, RCV, and other forms of voting as well. This is not surprising as voting is primarily mathematical in nature.

In the most natural sense, voting has a lot to do with data, specifically data about voters' opinions. In fact, each voting method can be broken up into two parts.

The first part is a voter opinion data collection method. This has to do with the type of ballot voters fill out. In plurality, only data about a voter's first choice is collected. In contrast, in an RCV election, voters are asked to rank all the candidates from most preferred to least preferred. Methods that collect information this way collect more data about a voter's opinion.

The second part is a voter opinion data interpretation method. This has to do with how to interpret the voter opinion data collected from the ballots. Plurality and IRV are example methods used to interpret this data.

Plurality simply checks which candidate has the largest number of first-place votes and declares that candidate the winner. Thus, it can be computed from data collected by single-choice ballots. However, the plurality winner could also be computed from a ranked choice ballot. To do this, we would only consider the most preferred candidate in each voters ranking and ignore the rest of the ballot. The candidate with the largest number of voters marking them as most preferred is declared the winner.

IRV instead considers each voter's complete ranking and as such requires a ranked ballot. In IRV, voter preference is determined in a round-by-round manner. In the first round, we only consider the most preferred candidate in each voter's ballot and check whether any candidate has a majority of votes. If any candidate does, then that candidate is declared the winner. This is very similar to what plurality would do if it were computed from a ranked ballot, except that in IRV, a true majority is required rather than simply the largest proportion of the votes. If no candidate has a majority of the votes, the candidate with the least votes is eliminated from every voter's ranking. Those votes are reallocated to the most preferred candidates remaining. This process continues until a candidate has a majority of the votes in a round, which is guaranteed to happen by at least the point where there are only two candidates remaining.

There are many other options for voting methods such as Borda count, Condorcet, ranked pairs, score voting, etc. Most considered methods are ways of interpreting ranked choice ballots. As such, it is a misnomer to refer to instant runoff voting as "ranked choice voting." Any voting method that uses a ranked choice ballot as its voter opinion data collection method could rightly be referred to as ranked choice voting. Since this discussion will explore mathematical properties of voting methods, we need to be more precise in our descriptions of voting methods. Thus, in this document, we will refer to instant runoff voting exclusively as IRV and not use the term RCV. For example, in this document, we will say that the Municipal Alternate Voting Methods Pilot Project initiated a pilot of allowing the use of IRV in local Utah elections.

One might note here that if we choose to rank order the candidates as the voter opinion data collection method, but use plurality to interpret those rankings, we might obtain different results. This is due to the idea that if we could only select one candidate, we might be more inclined to select the one candidate we liked best of those that we thought had a chance of winning as opposed to our actual favorite candidate. If we used a rank-order ballot, some voters might not realize that strategy is still optimal and so provide a more accurate ranking which might list a candidate as their most preferred that is different than the candidate they might pick for a single-choice ballot. What a voter decides to do may be even more different if we use IRV as the method to interpret the rankings. Voters may be more willing to vote for their actual preference first since they can list a less preferred but more likely to win candidate second and still feel like their voice is heard. The point here, though, is that both the voter opinion data collection method and the voter opinion data interpretation method made a difference in how a voter might choose to vote.

This is the other sense in which voting is primarily mathematical in nature. Specifically, in understanding how various voting methods affect the way that people make choices within them. This aspect of voting is in the branch of mathematics called game theory.

Many of the recent issues raised by citizens and some elected officials in Utah concern some of the game theoretical aspects of IRV. This report is intended to respond to some of these recently raised concerns about IRV, especially to make the mathematical concerns understandable to nonmathematicians, and discuss some of what mathematics can tell us about how we vote.

In this report, to aid in our response to various concerns about IRV including ones based in mathematical ideas, we first discuss briefly what it means that the mathematics of voting is a branch of game theory and why that concept is important to help us understand the differences between various election methods and how to decide which one to use. After that, we discuss the concerns raised about IRV which include some common concerns as well as concerns raised in a technical report by Jiri Navratil and Warren D. Smith (Navratil & Smith, 2022). Lastly, we conclude with a discussion about the pros and cons of both plurality and IRV considering what we know about them mathematically. To reduce the length of the main body of this report, we have moved some of the details of the mathematical discussion to the Appendix. There, we will delve more carefully into various voting fairness criteria, describe two additional voting methods for comparison, and describe two important mathematical theorems about voting methods.

Voting and the Mathematics of Decision Making

The mathematics of voting is part of a branch of mathematics called game theory. In a simple sense, game theory studies mathematically how players (e.g., people, campaigns, companies, animals, etc.) make decisions, how the rules of a game or other factors motivate players and change what strategies they will use to obtain the best outcome, and how to determine the optimal strategy within a particular game. A game is any strategic interaction between individuals that has a defined set of rules and some sort of objective or payoff.

What we commonly refer to as games in society, like board games or sports, are examples of such strategic settings, but many other situations also qualify. In economics, how companies compete and interact with each other is a mathematical game. In biology, how plants or animals compete for resources is a mathematical game. Many international relations, like war or diplomacy, can be modeled as a game. And in political science, how a society votes and campaigns is also a mathematical game. (For a more precise and rigorous introduction to game theory, see (Watson, 2013). For a nice and shorter resource on how game theory applies to voting, see (Wallis, 2014).)

One important observation about games is that the rules of the game determine the optimal strategy to achieve the objective of the game. In the context of voting, the voting method and other rules around how elections occur and who votes in which election all influence how voters will vote. This includes whether they will vote sincerely or strategically, how and where candidates campaign, and how political parties will develop and interact. Thus, when selecting a voting method to use, one should consider the implications of how those rules will affect the public's and the candidates' behavior surrounding the election. We explore this idea more extensively in the Appendix.

The Purpose of Voting

The question of what voting method to use is really a discussion of what kinds of behaviors each method's set of rules encourage and discourage, and whether such behaviors satisfy the purpose of voting. Recognizing that the objective of both candidates and voters in an election for a representative is to have a preferred candidate win the election and to avoid the election of less preferred candidates, we know that the strategies employed will be designed to maximize the chance that happens. Since a game's rules determine the optimal strategy, when we select the rules to voting, we should keep in mind the strategies that we want to incentivize and which strategies we want to discourage so the voting process accomplishes its purpose.

To do this, we specify here what the purpose of voting is and some notion of what strategies we should look to encourage. As suggested in the opening paragraph of this report, the purpose of voting for a representative is to accurately determine the collective opinion of the people about which candidate is preferred. We need to select an election method that, assuming honest ballots, selects as the winner the candidate that most reasonably represents what the voters collectively communicated. But if this is to be accurate, we also must incentivize honest voting as a strategy. That is, we should create rules that imply that a voter's optimal strategy is to submit a ballot with their actual preference listed. For the sake of peaceful interactions, it is also typically desired that campaigning be civil and as such, our voting method should also strive to incentivize civil campaigning as well. On the other hand, we should select rules that disincentivize strategic voting, that is, not voting one's true preference because they think that submitting an accurate preference would result in a less desirable outcome. Thus, we can summarize the purpose of voting and the goal of an election method as follows:

- **Purpose of voting and goal of an election method—**
    - ◊ to accurately determine the collective opinion of the people about which candidate is preferred.
- **Election method should incentivize—**
    - ◊ honest voting
    - ◊ civil elections

It is great that we can identify these big picture ideas of what an ideal election method does, but to determine whether a voting method does these things, we need to be more specific on what kinds of strategic voting we want to avoid. This has led to the construction of myriads of fairness criteria about voting methods. There are far too many such criteria to list, but in the Appendix, we discuss briefly about a dozen that are most commonly brought up in discussions about voting methods, some of which are directly or indirectly brought up in the concerns we address in this report. Note that perhaps the one thing that all the criteria have in common is that, at first glance, they all sound like desirable properties to have.

Fairness Criteria and Four Voting Methods

Here, we consider the four voting methods of plurality, instant runoff voting (IRV), ranked pairs (RP), and score voting (SV). Plurality can use single-choice or ranked choice ballots. IRV and RP are examples of methods that require a ranked-choice ballot. SV uses a score ballot. We discuss in this section which fairness criteria these methods satisfy and which they fail. We do this via Table 1, which indicates with a checkmark which criteria each of the four methods satisfy, and with an X which criteria they fail. For a more complete discussion of fairness criteria and a more detailed description of these voting methods, the reader is referred to the Appendix.

In Table 1, we consider the following fairness criteria, which we list with a brief, rough explanation of what the criteria require. More precise definitions are in the Appendix.

- The Majority Criterion—If a majority of voters rates the same candidate first, then that candidate wins.
- The Condorcet Winner Criterion—If a candidate beats every other candidate head-to-head, then that candidate wins. Such a candidate is called a Condorcet winner.
- The Condorcet Loser Criterion—If a candidate loses to every other candidate head-to-head, then the method cannot select that candidate as the winner. Such a candidate is called a Condorcet loser.
- The Clone Invariance Criterion—If two candidates are clones (politically speaking), then neither candidate affects the other candidate's ability to win.
- The Monotonicity Criterion—A candidate cannot be harmed by voters increasing their support for the candidate.
- The Strategy-Proof Criterion—A voter cannot improve their satisfaction with the results of the election by strategically voting; that is, a voter's best outcome is always obtained by submitting an honest ballot.
- The Independence of Irrelevant Alternatives Criterion—Whether an election method determines that one candidate is preferred to another is not affected by the presence of a third candidate in the election. This is effectively immunity to all kinds of spoiler candidates.

The above are not all possible fairness criteria, just ones that appear to be considered most often. (A more complete table can be found at (Wikipedia, 2024), but even that table is incomplete.) Also note that some fairness criteria are actually incompatible with each other, meaning that they cannot both be satisfied at the same time. Examples of these are the criteria listed in the two theorems in the Appendix, but there are other incompatibilities. Thus, it must also be determined which criteria are the most reasonable in deciding which is the best voting method.

Since practically every voting method, and certainly the ones we discuss here, satisfy the criteria of No Dictators, Pareto Efficiency, and Unanimity, which are rather simple and perhaps the most fundamental, we leave these criteria out of Table 1, but they are discussed in more detail in the Appendix.

	Voting Method			
Criterion	Plurality	Instant Runoff	Ranked Pairs	Score Voting
Majority	✓	✓	✓	X
Condorcet Winner	X	X	✓	X
Condorcet Loser	X	✓	✓	X
Clone Invariance	X	✓	✓	✓
Monotonicity	✓	X	✓	✓
Strategy-proof	X	X	X	X
Independence of Irrelevant Alternatives	X	X	X*	X*

Table 1. This table indicates which of several fairness criteria the four voting methods of plurality, IRV, RP, and SV satisfy. A checkmark indicates that that method satisfies that criterion, while an X indicates that it does not. The stars on a few of the Xs are explained in the body of the text.

For RP, we put a star by Independence of Irrelevant Alternatives. This is because while RP ultimately fails Independence of Irrelevant Alternatives by Arrow's Impossibility Theorem, it comes close. RP satisfies two weaker versions of Independence of Irrelevant Alternatives called Independence of Smith-Dominated Alternatives and Local Independence of Irrelevant Alternatives.

For SV, we put stars by Condorcet Winner, Condorcet Loser, and Independence of Irrelevant Alternatives to draw closer attention to them and make further comments. In the Appendix, we provide examples of how SV fails all three of these criteria, but point out that the situation is more complicated as there are assumptions on voter behavior that can affect whether SV satisfies these criteria or not. We refer the reader to the more thorough discussion in the Appendix. We will say here only that the required assumptions on voter behavior do not seem very realistic to these authors, and as such, it seems likely that SV fails all these criteria in practice.

Though more discussion on this topic is in the Appendix, we now have sufficiently described the main mathematical issues that are the substance of many of the concerns about IRV that we wish to consider. Thus, we now move to responding directly to those concerns.

Responses to Some General Concerns about IRV

During the IRV pilot program in Utah, there have been a number of questions raised concerning IRV and whether it is susceptible to various election issues. Many of these questions are about the game theoretic component of voting concerning how voters choose to vote and what strategies they might employ. One thing we note, though, is that if these questions only seem to interrogate properties of IRV, it might make plurality appear more attractive simply because the questions were not also applied to plurality. Thus instead, we will try to word these concerns so that they include a comparison to plurality. Some of these questions include:

- Does IRV cause more voter confusion and voter errors than plurality?
- Does IRV solve the spoiler effect that is observed in plurality?
- Does IRV or plurality fail the Condorcet Criteria?
- Does IRV or plurality fail the Monotonicity Criterion?
- Does IRV or plurality cause strategic voting?
- Does sequential IRV produce unfair outcomes when compared to traditional IRV or plurality?
- Does IRV result in different outcomes than plurality?
- Are ballots thrown out in IRV elections compared to plurality elections?

In this section, we strive to answer these questions mathematically.

Does IRV Cause More Voter Confusion and Voter Errors Than Plurality?

Since IRV employs a ranked ballot, which is somewhat more complicated than a single-choice ballot, it is natural to question whether IRV would produce more voter confusion and error than the current plurality system. Indeed, this has been an active topic of research. In fact, the authors responded to recent research on voting error in IRV elections criticizing the methods used and conclusions drawn (Parry & Kidd, 2024). As that research illustrates, great care is necessary to ensure that a discussion about voter error is not misleading. In that vein, there are a few things that are important to remember.

First, regardless of how complicated a system is, it is natural for there to be error and confusion when using it initially. Thus, comparing current voter error rates in IRV elections to voter error rates in plurality elections, when IRV has only been used for a handful of elections and plurality has been used in the USA for more than two centuries, seems likely to produce misleading results about the actual impact. One would expect that eventually, through common use, errors and confusion would decrease to a stable rate of voter error.

This is not dissimilar to learning curves associated with a new surgical practice. When new and more effective surgical procedures are first designed, practicing surgeons must learn the new techniques to incorporate them into their surgical practice. As with any skill, there are initially more surgical errors as those surgeons incorporate a new technique. That error rate decreases over time as the surgeon improves (Gofton, Papp, Gofton, & Beaulé, 2016). If the public was put off a new surgical practice because of an initial increase in error rate, they may miss out on the significant benefits provided by the new practice.

Similarly, any observed increase in ballot error rates associated with the transition to IRV is likely temporary and will decrease over time. To get a better understanding of whether IRV causes more voter confusion and error than plurality, we need to control for how long the system has been used and how exclusively it was used. Such a comparison does not seem to be currently available because IRV has not been used long enough or exclusively enough to make this claim. For example in Utah, the IRV pilot has only run for three election cycles. This seems like hardly enough time for voters to get used to the idea of filling out a ranked ballot without error. To fully understand the impact of any voting method on error, one would have to track error rates over multiple elections until it was clear that the typical learning curve has passed and error rates have settled.

Second, it is important to compare similar elections to each other. Error rates seem likely to fluctuate with more contested elections, larger numbers of candidates, how long the voting method has been used, levels of educational materials circulated about the voting method, etc. When making a comparison for whether it is the voting method that causes a difference in voting error or not, one must control for these other confounding variables.

Third, it is important to determine exactly what constitutes a voter "error." Some research seems to indicate that almost any behavior outside of completely filling out a ranked choice ballot with a candidate in every ranked position is an error. However, this can be a bit misleading. Just like a more complicated ballot can lead to more opportunities to do something by accident, it also allows for more opportunities to do something deliberate. Deliberately making a choice that is not mathematically optimal is not the same as making an error. There are three common ways that a ranked choice ballot can be unexpectedly filled out that are sometimes considered an "error." These are (a) over-ranking a candidate, (b) skipping rankings, and (c) over-voting a ranking. Over-ranking a candidate occurs when a voter indicates the same candidate at multiple ranking levels (e.g., ranking Candidate B 2nd, 3rd, and 5th). Skipping rankings occurs when a voter ranks a number of candidates at high-ranking levels, then does not indicate a candidate for one or more ranking levels before ranking another candidate below the levels skipped (e.g., ranking Candidate B and C 1st and 2nd, skipping the 3rd and 4th ranking position, and then ranking Candidate E 5th). Over-voting a ranking occurs when a voter indicates more than one candidate in the same ranking level (e.g., ranking Candidates A, B, and D all 3rd). As the authors pointed out in Parry & Kidd, 2024, the only one of these three that makes the ballot even partially unable to be processed is over-voting a ranking. While over-ranking and skipping rankings may, depending on the jurisdiction, technically violate the instructions given about how to fill out a ranked choice ballot, the IRV method can still process them as communicated. We argue that only filling out a ballot in a way that makes the ballot unreadable should be considered an error. Other choices could very well be a form of political expression. Those who count ballots should not be in the business of second guessing that political expression but should process the ballot as is.

Arguments that over-ranking and skipping rankings are errors seem typically based on the idea that such choices are non-optimal mathematically. Indeed, both over-ranking and skipping rankings provide no reliable mathematical advantage in an IRV election. However, the arguments that such choices are errors interpret this non-optimal choice as indicating that the voter does not understand the voting method. But if making a mathematically non-optimal choice indicates a voting error, then every vote for a third party in a plurality election should be considered an error as well, since that is a mathematically non-optimal choice. But it is unreasonable to consider voting third party an "error" as it is clearly a form of political expression. It is entirely possible for a voter to make a deliberate choice to do something that is mathematically non-optimal because of a political or moral opinion.

While some research has suggested that IRV increased ballot rejection rates due to error (Pettigrew & Radley, 2023), other studies suggest that the complexity of a ranked choice ballot has not led to an increase in errors that void a ballot (e.g., over-voting) (Kimball & Anthony, 2016). Experiments on this process have also shown that there is an increase in other choices that violate the instructions given, but not in a way that voids a ballot (e.g., skipping rankings or over-ranking) (Maloy J. , 2020) (Maloy & Ward, 2021). Another study of IRV in practice in San Francisco, where IRV has been implemented longer than anywhere else in the USA, showed that while ballot error rates were higher under IRV, the rates were low enough to not impact the results of the election, unless the election was very close, which is not typical (Neely & McDaniel, 2015). Considering all this together, it is unclear whether ballot error rates are generally different between IRV and plurality voting, but they do generally seem to be quite low.

One final thing to note here is that a discussion about ballot error rates must be made in an appropriate context. One of the main reasons that proponents of IRV support the measure is that it allows more people to participate in the meaningful choice between the main two candidates. In plurality elections, all votes for anyone other than the top two candidates are essentially ignored in the final tally which just compares the number of votes between the top two vote-getters. In some elections, the number of votes lost can be quite high. For example, in the 2020 Utah Republican gubernatorial primary, 28.9% of votes were lost this way (Office of the Lieutenant Governor, 2020). When discussing possible votes rejected due to increased error in IRV, it is important to balance that with a discussion of how many additional votes played a role in deciding who the representative is. If a tiny increase in error is coupled with a large increase in voters who play a role in deciding the representative, that is still a large net positive increase in voter influence. In which case, a small increase in error rates might still be preferable to the complete loss of all ballots voting for a candidate that is not a top-two candidate.

Does IRV Solve the Spoiler Effect that is Observed in Plurality?

The effect of a spoiler candidate in plurality is a well-documented phenomenon. Indeed, the 2020 Utah Republican gubernatorial primary, which we already mentioned, was so close between the two leading candidates that either the third or fourth place candidate had enough support that had they not run, the election potentially may have swung for Huntsman instead of Cox (Office of the Lieutenant Governor, 2020). Thus, either candidate possibly spoiled the election for Huntsman, though it depends on how those other voters would have voted if their chosen candidate hadn't run.

This is precisely the idea that proponents of IRV argue should be considered. The premise of IRV is to eliminate the least supported candidate and reallocate the votes for that candidate to those voters' next preferred candidates. Proponents for IRV contend that this eliminates the spoiler effect. However, recent research by Navratil and Smith has called that claim into question as they found that a recent IRV election in Moab, Utah experienced a spoiler (Navratil & Smith, 2022). In that election, Navratil and Smith noted that had one of the losing candidates not run, then it would have changed who won the election. Without the exact data, we cannot recreate precisely what happened in that election to verify this observation. However, when such spoiler effects happen in an IRV election, it is typically because removing the losing candidate from the election changes who is eliminated in each round, which also changes which votes are redistributed, resulting in different candidates and vote distributions in each round.

To unpack both the claim that RCV eliminates the spoiler effect and the claim that it does not, we need to be specific about what is meant by a spoiler candidate. A spoiler in a plurality election is typically considered a third candidate that is similar enough to one of the top two candidates that some of the voters that would have otherwise voted for one of the top two candidates instead vote for the third candidate. In a more fundamental sense though, a spoiler candidate is a losing candidate that changes the winner of the election simply by being in the election. This is precisely the effect that the Independence of Irrelevant Alternatives Criterion is referencing (see the Appendix for more information)—that the preference of a candidate by the electorate, which includes who the winning candidate is, should not be affected by whether some other candidate runs. As pointed out in the Appendix, this fairness criterion is incredibly difficult to satisfy. Indeed, practically no voting method satisfies this condition, except a dictatorship or similar system, which is the substance of Arrow's Impossibility Theorem for voting methods that can be done with a ranked ballot (see the Appendix).

Thus mathematically, IRV does not completely solve the spoiler effect, in that it does not entirely prevent the possibility that the presence of a losing candidate in the election always makes no difference in the selection of the winner. The effect observed by Navratil and Smith is real. But that is not surprising since practically no method is completely immune to spoilers. And the reader should also note that plurality, as we said, is also susceptible to spoilers.

However, IRV does prevent a certain kind of spoiler. Because spoiler effects are complicated, several fairness criteria target specific kinds of spoilers. The kind of spoiler that IRV does prevent is what is called a "clone." That is, IRV satisfies the Clone Invariance Criterion. This is the idea that if two politically identical candidates run, neither hurts nor benefits the other's ability to win. Plurality fails the Clone Invariance Criterion and indeed the idea of a clone is precisely the kind of spoiler that proponents of IRV are so concerned about. Plurality is incredibly susceptible to clones. In fact, plurality is an example of a strongly clone-negative method because the existence of a clone greatly decreases the chance of one of the clones winning. IRV does not share this effect as it is immune to clones. Thus, in the sense of this specific kind of spoiler (i.e., a clone), IRV does solve the spoiler effect.

Ultimately, IRV has more immunity to spoilers than plurality, but it is not entirely immune to all kinds of spoilers. Certainly, though, if the discussion is purely between plurality and IRV, then IRV is significantly better at dealing with spoilers than plurality. IRV certainly avoids the main type of spoiler that plurality is incredibly susceptible to.

Does IRV or Plurality Fail the Condorcet Criteria?

Of all the fairness criteria, probably widely considered the most important one is electing a Condorcet winner when there is one. Another similarly important criterion is avoiding electing a Condorcet loser. Even the paper by Navratil and Smith suggests that this is an important quality as they found that IRV failed to elect a Condorcet winner in a Moab, UT City Council Seat. They indicate that a Condorcet winner is the consensus candidate and that a voting method using a ranked ballot failing to elect the Condorcet winner is concerning (Navratil & Smith, 2022). Other authors refer to the Condorcet Winner Criterion as a "strengthening of the Majority Criterion" (Aazami & Bray, 2023) and so one would think that it would be important if majority rule mattered. And in a left-right political spectrum model of candidates, the Condorcet winner, if there is one, is the candidate that is most representative of the population (see the Median Voter Theorem and (Black, 1948)). There are also criticisms of the two Condorcet criteria, thus the reader should not take this as meaning that all mathematicians, political scientists, and economists agree that this is the most important criterion. If studying the problem of determining the optimal election method tells us anything, it is that it is extremely complex, and people even disagree on what is important in an election system. The authors of this paper agree with most that the Condorcet Winner and Loser Criteria are critically important in determining the effectiveness and fairness of an election method.

Notably, both plurality and IRV fail the important Condorcet Winner Criterion. Plurality also fails the Condorcet Loser Criterion; that is, plurality is capable of electing the Condorcet loser in an election.

Indeed, consider Hypothetical Election 1 between candidates A, B, and C described in Tables 5 and 6 in the Appendix. Candidate A is the Condorcet winner, winning both head-to-head matchups against Candidates B and C. On the other hand, Candidate C is the Condorcet loser losing both head-to-head matchups against Candidates A and B. However, if we interpret the preference profile using plurality, where we only look at the first-place winners, we find the results in Table 2 and plurality elects the Condorcet loser C.

Hypothetical Election 1			
# of Votes	30%	31%	39%
Order	A	B	C

Table 2. Plurality method applied to the preference profile for Hypothetical Election 1 in Table 6 in the appendix. In this election, plurality elects the Condorcet loser Candidate C. Note also that plurality misidentifies the Condorcet winner A as the worst performing candidate.

If we instead interpret the preference profile in Table 6 via IRV (see the Appendix for Table 6), then since no one has a majority in the first round, the candidate with the least first choice support is eliminated. In this case, IRV eliminates the Condorcet winner Candidate A, in the first round. The second round is then simply the runoff between B and C. This runoff is the third one listed in Table 5, in which B defeats C 51% to 49%. Thus, IRV elects Candidate B, which does avoid electing the Condorcet loser, but it fails to elect the Condorcet winner and eliminates a candidate that two-thirds of voters preferred to the winner (See Table 5 in the Appendix where Candidate A beats Candidate B in their runoff 67% to 33%).

Both plurality and IRV fail to elect Condorcet winners with some frequency. Of the 16 elections that Navratil and Smith considered, they found that all of them exhibited a Condorcet winner. IRV failed to identify the Condorcet winner only once, in the 2021 Moab City Council election for the first seat, while plurality failed to identify them twice, in the 2021 Moab City Council election for the first seat and in the 2021 Vineyard City Council election for the second seat (Navratil & Smith, 2022).

Ultimately, when it comes to the Condorcet criteria, neither IRV nor plurality have particularly great properties, which puts a dent in the idea that IRV always elects a consensus candidate. However, plurality is considerably worse in this regard. And using the Condorcet criteria to support plurality (which fails both) over IRV (which only fails the Condorcet Winner Criterion) is misguided. Overall, IRV is superior to plurality in its assurance to get closer to electing a consensus candidate. In fact, along with being incredibly susceptible to the spoiler effect, plurality's ability to elect a Condorcet loser is one of the strongest arguments against using plurality.

We note here that with the Condorcet winner being widely considered the consensus candidate, there are voting methods that guarantee electing them. Of the four methods we described in the Appendix (plurality, IRV, SV, and RP), only RP satisfies both Condorcet criteria. There are, however, many other methods that satisfy both Condorcet criteria.

Does IRV or Plurality Fail the Monotonicity Criterion?

The Monotonicity Criterion is an interesting one. The idea that increasing a candidate's support could potentially hurt that candidate's chances of winning is so paradoxical that it seems odd to even have to talk about it. However, IRV is one type of voting system that is susceptible to this possibility.

For example, consider the following election between candidates A, B, and C. In a ranked choice voting election, voters submit a ranking of the candidates. In Table 3 below, we list what portion of the electorate ranked these three candidates in each ranked order. For example, the first column indicates that 16% of voters ranked the candidates in the order A first, then B, and C last. A table like the one in Table 3 that lists how the entire electorate ranked the candidates is called a preference profile.

Hypothetical Election 2						
# of Votes	16%	18%	14%	20%	20%	12%
Order	A B C	A C B	B A C	B C A	C A B	C B A

Table 3. Preference profile of Hypothetical Election 2 resulting from ranked choice ballots in a race between three candidates A, B, and C. This election exhibits IRV's susceptibility to failing the Monotonicity fairness criterion. Candidate A wins this election via IRV.

For first choice votes, Candidate A has 34% support (16% from the first column and 18% from the second column). Candidate B also has 34% support, while Candidate C has 32% support. Since no candidate has a majority, by the IRV method, the candidate with the least first place votes is eliminated. In this case, this means that Candidate C is eliminated. This redistributes their votes, with 20% of voters going to Candidate A and 12% going to Candidate B. Then in round 2, Candidate A defeats Candidate B 54% to 46%.

Now suppose that rather than the actual election, Table 3 only indicated the preferences of the voters a few days before the election. No one actually knows this data, but suppose that Candidate A is aware that Candidate B is likely their strongest competitor. Thus, in the last few days before the election, Candidate A worked hard to convince 3% of voters to switch their ranking from B>A>C to A>B>C resulting in the revised preference profile in Table 4, which is what the electorate submits on election day.

Hypothetical Election 2-1						
# of Votes	19%	18%	11%	20%	20%	12%
Order	A B C	A C B	B A C	B C A	C A B	C B A

Table 4. Preference profile of Hypothetical Election 2 resulting from ranked choice ballots in a race between three candidates A, B, and C, but after Candidate A increased their support among the B>A>C voters by 3%. As a result, candidate C wins this election via IRV. This means that Candidate A made themselves lose the election by increasing their support.

In this revised election, Candidate A now has 37% first choice support, Candidate B has 31%, and Candidate C still has 32%. This time then, Candidate B is eliminated by IRV. This redistributes their votes so that in round 2, Candidate C now defeats Candidate A 52% to 48%. Essentially this means that by increasing their support, Candidate A made themselves lose the election. Even more concerning is that if an electorate's preference profile was such that IRV would fail monotonicity, this would likely happen in an undetectable way since no one knows the preference profile of the public until after the election, meaning that Candidate A would have no idea how much support they needed before a win would turn into a loss. On one hand, the undetectability of a monotonicity failure before the election cuts against the idea that someone could use this feature of IRV strategically, but it also means that a candidate could inadvertently make themselves lose by doing an activity that is generally considered helpful (i.e., increasing their support).

Note that while this is possible in an IRV election, it will not happen every time. For example, Navratil and Smith note that if three specific voters out of 1803 voters in the 2021 Moab Council Seat 1 election had increased their support for the winner, then that winner would have lost via IRV (Navratil & Smith, 2022). But this was the only monotonicity failure found in the sixteen races they considered. It is notable, though, for how close the winning candidate was to losing the election for themselves by garnering more support.

On the other hand, this is one of the few fairness criteria that plurality satisfies. In fact, monotonicity is close to the only fairness criterion where plurality succeeds but IRV fails (the Participation Criterion is the only other example the authors are aware of).

Does IRV or Plurality Cause Strategic Voting?

Strategic voting is the act of a voter not voting their honest opinion because they believe that a better outcome will occur if they vote otherwise. Whether IRV or plurality causes strategic voting is an interesting question, but one to which we have a mathematical answer. By the Gibbard-Satterthwaite Theorem and Arrow's Impossibility Theorem (see the Appendix for the statements of these theorems), every voting method that uses a ranked choice ballot is susceptible to strategic voting. Thus, in its simplest sense, the answer is yes, both IRV and plurality cause strategic voting. However, it does not mean that they cause the same kind of strategic voting or that the prevalence of strategic voting is the same.

Strategic voting is remarkably prevalent in plurality. The notion that one must vote for one of the two major party candidates, because to not do so is "throwing your vote away," is strategic voting. Feeling compelled to "vote for the lesser of two evils" is strategic voting. In fact, strategic voting abounds so much in plurality that a voter voting their mind seems odd, especially if the voter's favored candidate is a third-party candidate. This means that the results of plurality elections are unreliable indicators of public opinion. The nature of plurality fundamentally undermines the ideal of incentivizing honest voting that we identified as a main goal of a voting method. As a result, plurality essentially fails the purpose of voting since it cannot accurately determine the collective public opinion of the people while so strongly disincentivizing voters from providing an accurate depiction of their own opinion.

In fact, the incentive to not support trailing candidates is so strong that it has been identified as the major cause of the two-party system. Duverger's Law indicates that within single member districts that use plurality voting systems, two main parties eventually emerge (Duverger, 1954). This law is based on empirical evidence, but there is a mathematical proof under certain assumptions in the limit as the electorate gets large (Palfrey, 1989).

IRV also produces some strategic voting, but not in the same way and certainly not to the same degree. On the contrary, the fact that IRV considers an entire ranking of all the candidates from each voter allows the voter considerably more freedom to express their political opinion. Indeed, every ranked ballot voting method provides this same freedom to voters. Moreover, how IRV determines the winner from a preference profile is designed to minimize any drawback of identifying a voter's true preference. This mechanism is specific to IRV as other ranked choice voting methods, like RP, determine the winner differently.

While strategic voting in IRV is possible, many of the strategies would require more complete knowledge of the electorate's preference profile than would be available prior to the election. Strategic voting of that type, while technically possible, is not particularly relevant in practice since a voter cannot know how to act on knowledge they do not have and hence would be incentivized to still vote honestly. For example, voters could theoretically use the fact that IRV fails the Monotonicity Criterion to manipulate who is eliminated in the first round, but to do so would require knowledge about how the rounds would result prior to the manipulation, which is information that the voter will not have prior to the election. As such, the effect of monotonicity failure on strategic voting is minimal as the best strategy a voter has, given the available knowledge of the electorate's preference profile, is to vote honestly, at least in the context of taking advantage of a monotonicity failure.

Meaningful strategic voting depends on how voters strategize with the knowledge available to them prior to the election. For example, in a plurality election, a voter may perceive that a tie is likely between the two candidates with the greatest support and that their most preferred candidate, which is not one of those two, is likely to lose. This can incentivize a voter to insincerely indicate that their preference is for one of the candidates with the greatest support in order to affect the potential tie between them. This is the main driving force behind the idea of "voting for the lesser of two evils."

A voter's belief in the likelihood of a tie between candidates may also influence them to insincerely vote in an IRV election. In this case, a voter in an IRV election might choose to not list their favorite candidate first in order to affect a potential tie in the first round, but there is no advantage to ranking their favorite candidate any lower than second. A similar strategy also occurs in a regular runoff election and the strategy is perhaps a little more obvious there. In a runoff election, a voter that supports A, but believes that A has strong enough support to advance, might cast their first vote for candidate B if they believe that candidate A has a stronger chance of defeating B than another candidate. In this case, the voter is using their first-round vote effectively to vote for the opponent they want to run against their preferred candidate. That voter then switches their vote to Candidate A in the runoff. While this effect could still provide some benefit in an IRV election, it is critically different. In IRV, the preference is locked when the voter submits their ballot. Thus, the voter's vote can only switch to their preferred candidate, if listed second, if the candidate they listed first is eliminated. Thus, there is considerably less incentive to use this strategy in IRV than in a regular runoff election.

Ultimately, the situations where strategic voting may help are more complex in IRV than in plurality, but beyond that, the payoff, and hence the incentive, is also typically much smaller. A recent analysis comparing the susceptibility to strategic voting in both IRV and plurality found that while more voters may be able to benefit from voting strategically in IRV than plurality, the payoff compared to plurality is much smaller. In fact, the benefit to the average voter of taking strategy into account is many times larger in plurality than in IRV (Eggers & Nowacki, 2024). Thus, the incentive to vote strategically is smaller in IRV and hence it performs better at encouraging the electorate to vote their honest preferences than plurality does. That is, IRV is less likely to produce strategic voting on the whole than plurality is.

Does Sequential IRV Produce Unfair Outcomes When Compared to Traditional IRV or Plurality?

In many city council elections, multiple seats are up for election at the same time. Typically in Utah, the entire candidate pool runs concurrently and the election method selects the top candidates to take the seats. In non-IRV elections, this is done with a vote-for-n ballot where n is the number of seats available, then the n candidates with the most votes are elected to the seats. This presents a challenge for IRV since it only elects a single winner and does not produce an overall preference ordering of the candidates.

There are a number of options one can do to have IRV identify more than one winner. For example, if there are only two seats available, then IRV could simply elect the two candidates that survive to the final round. That is, the winner would take the first seat, and the loser of the last round would take the second seat. However, for the second seat winner, this method fails to consider the opinions of the voters whose votes are currently occupied electing the first-place winner. The entire point of having voters "vote-for-n" in a multi-seat election is to take the entire electorate's opinion into account for each seat. Moreover, this method also would not work if a city needed to elect more than two seats. Thus, the way IRV is typically employed to fill multiple seats is to use it sequentially. That is, IRV is run to elect the winner to the first seat. Then that candidate is removed from the preference profile and those that voted for them now have their preferences redistributed to their next choice candidate. Then the IRV process is run again, and the winner of that process is elected to the second seat, and so on. This process has the paradoxical effect that the winner of the second run of IRV is not necessarily the losing candidate that survived to the last round in the first run. Indeed, Smith and Navratil observed that this happened almost half the time in multi-seat city council races in 2021 in Utah (Navratil & Smith, 2022).

Some argue that this outcome is unfair. But it is no more unfair than using a single-choice ballot to elect a multi-seat election. Suppose two seats were available in a city council election and the city only allowed voters to select a single candidate. Then the city determined the winners by selecting the candidates with the two largest number of votes. But the second seat winner in that scenario might not have received the second most votes if voters were allowed to select two candidates. Thus, plurality also runs into this problem. Again, the reason why a "vote-for-two" method would be used in this scenario is so that the entire electorate's opinion can be considered for both seats. We would expect that most would consider a "vote-for-two" method fairer and more appropriate in a two-seat city council election than a single-choice plurality method. Similarly, sequential IRV is much fairer and more appropriate than simply selecting the second-place winner in the first IRV run to take the second seat since it takes the entire electorate's opinion into account.

Between sequential IRV and selecting the last round loser as the second seat candidate, sequential IRV is the method more similar to the vote-for-n modification to plurality that is typical in multi-seat non-IRV elections. Thus, for consistency, if one argues that vote-for-n is fairer than single-choice plurality to decide a multi-seat city council election, they should also support sequential IRV to decide a multi-seat city council election. The so-called "second place winner misidentification" paradox is a red herring when comparing plurality to IRV, since both systems are susceptible to it, and in both cases, the modification to the typical method that causes it is intentional and meant to consider the entire electorate's opinion for each seat.

Does IRV Result in Different Outcomes Than Plurality?

One of the most important ways to evaluate an alternative method to a more traditional one is to consider when they disagree. If an alternative never disagrees with the original, then it is not particularly useful in effecting change. On the other hand, if, when an alternative disagrees with the original, it does so in an undesirable way, then it is not likely a good alternative. Thus, it is natural to ask whether IRV and plurality ever disagree.

Plurality and IRV disagree sometimes on who the winner is, though not often. Navratil and Smith observed that among the sixteen IRV races in Utah in 2021, IRV and plurality disagreed once, in the 2021 Vineyard City Council election for the second seat (Navratil & Smith, 2022). IRV picked the Condorcet winner while plurality did not. This supports the idea that IRV is better at identifying the consensus candidate, though as we said previously, it does not always do this.

But the situation is more complicated than what Navratil and Smith indicate. The comparison between IRV and plurality that Navratil and Smith made is not a true comparison between the typical practical use of these methods. They identify the plurality winner in those elections by which candidate had the largest amount of first-place support on the ranked choice ballot. However, we have already pointed out that the rules of the game determine the strategy. As such, which candidate voters list first in their rankings in an IRV election might not coincide with whom they would select in a plurality election with a single-choice ballot. Indeed, it is a main argument of proponents of IRV that many voters would choose different candidates. As such, considering the first-place votes in an IRV preference profile is not the same, in practice, as considering an actual plurality election. A true comparison between IRV and plurality is impractical as it would require voters to vote twice; once in an IRV election and again in a single-choice plurality election. This might be doable in polls but would be neither practical nor appropriate in a real election. Thus, we cannot truly determine how often plurality and IRV actually disagree. We think that the likelihood of the two methods disagreeing is greater when making a true comparison since in a true plurality election, the incentive is much larger for a voter to change their first-place vote to a candidate that is not actually their first choice than it would be in an IRV election for the same race.

Ultimately, it is true that IRV and plurality disagree sometimes, and when they did in the 2021 election cycle, IRV was better at identifying the consensus candidate than computing the plurality winner from the IRV-ranked ballot.

Are Ballots Thrown Out in IRV Elections Compared to Plurality Elections?

Exhausted ballots are a feature of IRV that has caused some confusion. A ballot becomes exhausted in an IRV election when all the candidates ranked on that ballot have been eliminated. Critics of IRV erroneously argue that when this happens the IRV process has thrown the ballot out. However, the ballot is not thrown out, it simply no longer contains any relevant information.

This is no different than a vote for a third-party candidate in a plurality election. How many people voted for a candidate other than the top two is irrelevant in the comparison of the top two candidates to determine which received more of the vote. Those third-party votes are just as non-influential as an exhausted ballot. This phenomenon is discussed at length in a recent joint report between FairVote and the Gary R. Herbert Institute for Public Policy (Hutchinson & Parry, 2023).

Overall, it is not true that IRV elections throw out votes. Ballots in IRV elections are only rejected for similar reasons as those rejected in plurality elections, which is only for an error that makes the ballot undecipherable. Exhausted ballots are removed from consideration because they no longer contain voter preferences for non-eliminated candidates, but they are always recorded and included in the determination as much as their provided information allows. If voters are concerned that their ballot will be ignored after a certain round in an IRV election, they can avoid this by ensuring that they fill out a complete ranking of candidates. When a ballot is exhausted because a voter chose not to submit a complete ranking, it is an artifact of that voter's political expression and not a failing of the IRV system.

Responses to Concerns Raised by Navratil and Smith

In 2022, Jiři Navratil and Warren Smith published an analysis of the 2021 IRV elections in Utah (Navratil & Smith, 2022). They point out many potential failings of IRV in those recent elections and this report and others have been used by critics of IRV in Utah. (See (Davidson, 2024) for an example where three of the four references Davidson cites on his webpage for why he believes "the theory falls apart" come from Navratil.) Those that use this work to criticize IRV are often doing so in favor of maintaining the status quo single-choice plurality method.

The most important problem with using Navratil's and Smith's work against IRV and in favor of plurality is that neither Navratil nor Smith are proponents of plurality. In fact, Smith opposes plurality on his website promoting his preferred voting method SV (a.k.a. range voting) (Smith, 2005). Using Navratil's and Smith's work to push for plurality over IRV is a naïve and disingenuous position that misrepresents what Navratil and Smith are trying to say.

The main point of Navratil's and Smith's paper is to argue that in addition to plurality being a poor election method, IRV is poor too. They claim that instead of arguing between those two bad methods, we should be looking at different methods entirely. And mathematically, they are not wrong in this point. As we discuss in the Appendix, there are many other kinds of voting methods, and while IRV is generally superior to plurality, several of these other voting methods have superior qualities to either plurality or IRV. It would be good to explore other ideas, and to allow the public the freedom to explore those other ideas as they see fit in their local elections and perhaps on larger stages too.

That said, in this section, we will explore some of the claims made by Navratil and Smith in their analysis and work to clarify what they are observing. In the remainder of this section, for brevity, we will refer to the Navratil and Smith paper as the NS paper. We first comment briefly on four criticisms made in the NS paper that we have already addressed more carefully in other sections. Then we respond to the following remaining questions brought up by the criticisms and observations made in the NS paper.

- Does IRV elect a majority winner?
- Does IRV fail the Participation Criterion? Does it matter?
- Does IRV produce voting paradoxes? Does it matter?

Items Addressed in Previous Sections

There are a handful of claims made in the NS paper that we discussed in the previous section on general concerns. These include the following four observations presented in the NS paper.

The NS paper indicated that standard IRV misidentifies the second-place winner. This is important in a multi-seat election that will use IRV. Suppose that IRV is run in a ranked choice election and a winner identified. If that winner is removed from the ballot and the IRV process is run again, then the candidate who appeared to come in second the first time is not necessarily who wins in the second IRV run. It is true that this happens with some frequency in IRV, but there is a good reason to choose for the second seat the winner of the second iteration using a second running of IRV rather than just selecting the runner-up of the first run. We discuss this in more detail above in the section "Does Sequential IRV Produce Unfair Outcomes When Compared to Traditional IRV or Plurality?" and the reader is referred there for more discussion.

Second, the NS paper observed that IRV fails the Monotonicity Criterion in their discussion concerning the 2021 election for Moab City Council. This observation and what it means for IRV to fail the Monotonicity Criterion and whether that is important is discussed in the section above on "Does IRV or Plurality Fail the Monotonicity Criterion?" and the reader is referred there for more discussion.

Third, the NS paper determined that IRV fails the Condorcet Winner Criterion. This is also a feature of plurality. The NS paper observed that IRV did not elect the Condorcet winner in one of the sixteen elections they considered, and plurality failed to elect the Condorcet winner in two of those same elections. Each of the sixteen elections had a Condorcet winner. We discuss the fact that IRV and plurality do not always elect the Condorcet winner as well the fact that plurality sometimes elects the Condorcet loser in the section "Does IRV or Plurality Fail the Condorcet Criteria?" and in the section "Fairness Criteria and Four Voting Methods" and the reader is referred to those sections for further discussion. We will, however, note here that the NS paper criticizes IRV for failing the Condorcet Winner Criterion and calls it "concerning," while at the same time the authors promote SV, which, as we show in the Appendix, fails both the Condorcet Winner and Condorcet Loser Criteria (see the Appendix for an example election where this happens).

Fourth, the NS paper noted that IRV is not entirely immune to spoiler candidates. In the NS paper, they observed a spoiler in the 2021 election for Moab City Council. As we note in the section "Does IRV Solve the Spoiler Effect that is Observed in Plurality?", there are several types of spoilers and practically no election method is entirely immune to spoilers (see the sections in the Appendix "The Independence of Irrelevant Alternatives Criterion" and "Two Most Unfortunate Mathematical Facts About Voting"). We also note here that the NS paper criticizes IRV for failing to prevent spoilers, while the authors promote score voting which also fails to prevent spoilers (see the Appendix for an example election where this happens). However, this is perhaps not as problematic as the Condorcet criticism since practically all voting methods fail to prevent some type of spoiler. More notable in their criticism is the lack of reference to Clone Invariance that IRV does satisfy, which omits an important element of this discussion that IRV prevents the main type of spoiler observed in plurality and hence the argument that IRV prevents spoilers has meaningful validity.

Does IRV Elect a Majority Winner?

One observation made in the NS paper is that in three of the sixteen IRV races they considered, the number of votes for the winner in the final round was not a majority of total votes cast. It is true that the majority of votes in the final round of an IRV election is not always a majority of votes cast. This is, however, entirely due to people in the electorate turning in an incomplete ranking. If every voter filled out a complete ranking, then the majority in the final round would always be a majority of votes cast, since all would still be included.

When a ranked ballot in an IRV election only includes votes for candidates that have been eliminated, there is no relevant information remaining on the ballot. By not ranking all the candidates, the voter has communicated that if the election no longer includes any of the candidates that they ranked, then they have no preference in who is elected, and they no longer want a say. This could very well be the intended political expression since, for example, many voters feel that they should not place a vote for any candidate that they would not approve of being in office. Filling out an incomplete ranking in this way would be the same as turning in an empty ballot if the election were only between the candidates they did not rank. Such a blank ballot would not be considered part of the ballots cast in determining a majority in any system. Each successive round includes all voters who indicated that they wanted a say in selecting the winner if the election were down to those candidates and as such, IRV produces a majority winner among the voters that care about the outcome between the remaining candidates. Thus, IRV elects a majority winner in the only meaningful way possible given voter choice.

Moreover, this complaint about IRV, especially when it is made to support either plurality or SV, seems insincere. Plurality does not guarantee a majority decision of any kind, as the name of the method itself expresses. Further, SV fails the Majority Criterion, meaning that it is possible in an SV election for the majority of the populace to favor A to B and SV still elects B. It is illogical to criticize IRV for failing to obtain a majority of all votes cast while simultaneously supporting plurality or SV when those methods are fundamentally designed to allow significant non-majority winners.

Does IRV Fail the Participation Criterion? Does it Matter?

Voting paradoxes involving participation are curious. These are situations where one of two things happen:

1. There is a set of voters who voted, but if they had chosen not to vote, then a candidate that they ranked higher than the winner would have won instead.
2. There is a set of voters who did not vote, but if they had chosen to vote, then a candidate that they ranked lower than the winner would have won instead.

In both scenarios, if the set of voters had perfect knowledge of the outcome of the election or the current preference profile of the electorate, they would be incentivized not to vote. However, those voters must have perfect knowledge of the current preference profile of the electorate before that electorate casts their votes to know that this strategy is even available. That said, if an election method can potentially produce one of the two situations above, it is said to fail the Participation Criterion (that is, the Participation Criterion states that an election method should not allow either of the two scenarios above).

In the NS paper, they noted that a participation failure was detected in one out of the sixteen IRV races they studied. Aside from the fact that this indicates that participation failures are likely rare, it also indicates that IRV indeed fails the Participation Criterion. However, looking closer at how IRV failed the Participation Criterion in this race provides a bit more insight.

The race in question was the 2021 Moab City Council race, which included 1803 voters and six candidates running for two seats decided via sequential IRV. The IRV run where the participation failures were detected was the first run to determine which candidate was elected to the first seat. Navratil and Smith detected two participation failures in this race.

The first showed that among the 1803 voters, if three specific voters who ranked the winner of that race last had stayed home and not voted, then a different candidate would have won, specifically the candidate that all three of these voters listed as their second choice. This certainly seems, on its surface, somewhat concerning and certainly those three voters likely regret the choice of voting that day. However, there is no way that these voters could have known that their votes cast would have behaved that way, especially since the impact of their votes depends heavily on how everyone else voted too. As such, there is no way that a voter could use this information to strategically change their ranking or decide not to show up and vote. And certainly, if only three out of 1803 voters created a participation failure, these voters should see that they are much more likely to influence the election the way they intend by participating than they could influence it in another way. As such, rational voters should not be discouraged to vote by this information.

The second participation failure is less convincing. Navratil and Smith remark that if 765 additional voters had all voted with a very specific ranking that ranks a particular candidate last, then the candidate they listed last would win. While this artifact is perhaps interesting intellectually as a quirk, it is incredibly unrealistic. First, the number of voters who voted in the race is 1803. Adding 765 additional voters is more than a 42% increase in the number of voters. Moreover, the city of Moab only has about 5300 people in it. That means almost an additional 15% of the population would have had to vote and all of them would have to rank the candidates in exactly one specific way. There are 720 different ways to rank six candidates, and that's only counting complete rankings. The idea that 15% of the population of Moab was sitting at home on election day all having the same ranking of candidates in mind out of 720 possibilities seems far-fetched. The authors of the NS paper likely included this example since it was a mathematical observation they made, but it should be noted that this example is remarkably unrealistic and not very meaningful as a result.

As stated previously, participation failures will never affect strategic voting because it requires information that is not available until after the election. And even then, detecting participation failure for many commonly used voting systems is NP-hard (see (Mohsin, et al., 2024)). While IRV was not one of the methods considered in this study, it does seem to indicate that even obtaining this information after the election is exceedingly difficult. Thus, the rare occurrence of a participation failure in an IRV election would have very little effect on whether a voter decided to vote or decided to vote sincerely.

Moreover, the Participation Criterion is designed from the point of view of the voter playing the voting game with their own priority of obtaining the best result for themselves. When we are deciding as a group which election method to use, we should be using the goal of the entire group, which is to obtain a voting system that encourages the electorate to vote honestly and interprets those votes in as accurate a way as possible.

The bigger issue with participation failures is that they indicate some kind of potential logical failing inherent in the election method. But no electoral system ever devised is void of some kind of logical issue. Thus, one has to balance the potential failings of each system with its benefits. Since participation failures require information that is not available to voters until after the election, it cannot affect how people vote or whether they vote. Moreover, more participation is always preferable for the collective goal of accurately representing the electorate, thus whether a voter inadvertently harms their own personal objective by participating is not particularly important to the entire group if the representation of the collective opinion is still accurate. Thus, the satisfaction of the Participation Criterion is, in the opinion of these authors, of limited value and it is certainly less important than other criteria or features that do affect how people vote or are more fundamental to whether the result is representative of the populace such as Clone Invariance, the Condorcet Criteria, or what kind of ballot is used.

We should note here that certain methods do satisfy the Participation Criterion, most notably, plurality and SV. Other methods such as a Borda count or curiously selecting a random ballot to determine the winner satisfy the Participation Criterion as well. IRV fails it, but ranked choice voting methods that satisfy the Condorcet Winner Criteria satisfy the Participation Criterion in the case of three candidates, but fail the criterion in the case of four or more candidates (Moulin, 1988). In fact, this last point shows that to satisfy the Participation Criterion, the method must fail the Condorcet Winner Criterion for four or more candidates. The Condorcet Winner Criterion is generally considered the most important criterion to satisfy and as such, participation failures seem like a necessary evil. Fortunately, recent research indicates that participation failures are likely incredibly rare. A 2023 study showed that of a data set of 315 elections using a ranked ballot, no participation failures occurred for methods similar to ranked pairs (Mohsin, et al., 2024).

Ultimately, IRV's failure to satisfy the Participation Criterion is perhaps worth noting, but it is not worth worrying about. The Participation Criterion is failed by many voting systems, is a very rare occurrence even when it is possible, contradicts more important criteria like the Condorcet Winner Criterion, and is incredibly difficult to detect let alone use in any strategic way. As such, it holds very little bearing on whether a person votes or how they vote and has limited value in determining whether a voting method is viable or not.

Does IRV Produce Voting Paradoxes? Does it Matter?

The main purpose of the NS paper was to analyze the 2021 IRV elections in Utah and identify any paradoxes that occurred. This is useful in determining how often various voting paradoxes occur, which can help guide a legislature in determining which voting methods to use. However, it must be kept within the proper context. While the NS paper shows that IRV exhibits various voting paradoxes from time to time, it is important to realize that all election methods produce paradoxes, including plurality. The existence of paradoxes within IRV is far from unique.

Knowing that all election methods produce paradoxes, the task of selecting which method to use again should be done considering the goal of a voting method, which is encouraging voters to vote honestly and being able to accurately determine the will of the people. Not all voting paradoxes are of equal relevance to that goal.

Thus, it does matter that IRV produces voting paradoxes, but it also matters that plurality, SV, and RP do as well. However, it is in comparing how those voting paradoxes affect that method's ability to achieve the goal of a voting method that matters. And using the NS paper to claim that only IRV produces voting paradoxes, or that its lack of ability to avoid them is somehow unique, is misguided and naïve.

Discussion and Conclusion

In this report, we have addressed the game theoretical issues underlying many concerns about IRV. While it is true that IRV falls victim to various voting paradoxes and fails certain fairness criteria, so does every election method. In general, on almost every mathematical consideration, IRV fares better than plurality (with a notable exception for monotonicity). Thus, the mathematics of voting tends to favor IRV over plurality.

We have also discussed what the purpose of voting is and what makes a good voting method. It is useful to reiterate that here. The purpose of voting is to accurately determine the collective opinion of the people about which candidate is preferred. And the goal of any election method is to accomplish that by encouraging honest voting and civil elections. In this report, we have focused our attention more on how voting methods accurately reflect the people's collective opinion and how they encourage honest voting. An entirely different discussion is how voting methods would affect how candidates run their campaigns or how political parties form and lead coalitions, but that will need to be the subject of other reports or papers.

While the purpose of this paper is to clarify the substance behind concerns about IRV and attempt to resolve those concerns wherever possible, we also discussed where IRV and plurality sit in the wider discussion about voting methods. We agree with IRV critics Navratil and Smith that it is worth having more discussion about which voting methods work best, and that neither plurality nor IRV is the optimal answer. However, one major advantage to continuing to explore IRV is the fact that it will help the populace be more open to the ideas of methods with superior mathematical qualities such as RP. Moreover, since RP is also a ranked choice voting method, IRV already uses the same kind of ballot.

Mathematicians who are informed on the mathematics of voting seem to agree with proponents of IRV that plurality is generally worse than IRV, and that keeping the conversation about voting focused solely on plurality is not good for democracy. Supporting change that allows for more discussion on this idea is preferable, and pushing for IRV support is the optimal way, at the present time, to encourage more robust discussion. Thus, opponents of plurality, whatever their preferred voting method is, should support the move to IRV as part of pushing for a larger discussion, since IRV appears currently to have the strongest support of any alternative voting method.

Appendix: Mathematics and Voting

In this Appendix, we delve deeper into the mathematics of voting. We first discuss an important principle of game theory and how that applies to the purpose of voting and the goal of an election method. Next, we describe fairness criteria that are designed to ensure that an election method achieves the purpose of voting. We also describe two election methods besides plurality and IRV, namely, ranked pairs (RP) and score voting (SV). We do this to compare their properties to both plurality and IRV, and to show that the discussion of voting methods is perhaps quite larger than what is currently occurring in the public sphere. We also present two well-known mathematical theorems about voting that contain some unfortunate news about the effectiveness of various voting methods. But these theorems do provide some insight into what to look for in a voting method and that discussion is how we conclude the Appendix.

The Rules of a Game Determine the Optimal Strategy

We described in the body of this report that an important observation about games is that the rules of the game determine the optimal strategy to achieve the objective of the game. In fact, in modern sports, the official rules are often changed in order to change the strategies that players employ. Consider a couple of examples from basketball.

In the late 1960s, the American Basketball Association (ABA) added the three-point line to make the game more enjoyable for fans and compete with the NBA. It quickly changed the way players played and coaches coached. Prior to the three-point line, the optimal strategy for scoring was to get the ball inside for higher percentage shots closer to the basket. With the incentive of the three-point line and the risk that some players could make shots from that far away, it changed the optimal strategy both on offense and defense to try for and defend against shots inside and out. This spread the players out more and made for a more entertaining game. Even today, players are improving their ability to take this higher reward shot and the game continues to be affected (USA Basketball, 2013).

A more recent example is the changes made to the NBA's rules for the 2023–2024 season. These included a rest rule that disqualified players from major awards if they did not appear in at least 65 games, and a flop rule that made embellishing and acting as though a player had been fouled a more severe foul for the flopper. These rules were put in place to encourage great players on good teams to keep playing even if their team secures a playoff spot early and to discourage players from faking or exaggerating being fouled (Anderson, 2023).

In both the examples above, the purpose of playing the game is also important. From the old ABA's or the NBA's perspective, their purpose in facilitating the professional play of basketball is to make money from fans. As such, their goal in determining the rules is to make the game more enjoyable for fans so that fans will spend more money on the games. They select the rules of the game to incentivize player strategies that are more conducive to their overall purpose of making money.

Just how the rules of a sport determine the optimal strategy for achieving the sports objective, the rules surrounding how people vote affect the optimal strategy not just for candidates campaigning, but also for how voters choose to vote. The rules concerning how we vote are highly complex and include everything from voter registration, to redistricting (and hence gerrymandering), to the electoral college. The voting method we use, however, is one of the most important parts of that set of rules and what the optimal strategies are for both the voters and the candidates.

For example, using single-choice ballots and plurality voting in the presence of two strong political parties determines that the optimal strategy for the voter is typically to vote for a candidate nominated by one of these two parties regardless of whether the voter finds either of those two candidates desirable as a choice. Hence the common voting mantra of "voting for the lesser of two evils." The rules of single-choice plurality incentivize this kind of strategic voting over honest voting.

In the next section, we discuss in detail various fairness criteria that are designed to ensure that a voting method encourages the behaviors of both voters and candidates that are conducive to the purpose of voting.

Fairness Criteria

It is great that we can identify these big picture ideas of what an ideal election method does, but to determine whether a voting method does this, we need to be more specific on what kinds of strategic voting we want to avoid. This has led to the construction of myriads of fairness criteria about voting methods. There are far too many such criteria to list, so we will discuss briefly about a dozen that are most commonly raised in discussions about voting methods, some of which are directly or indirectly brought up in the concerns we addressed in this report. Note that perhaps the one thing that all the criteria have in common is that, at first glance, they all sound like desirable properties to have.

The Majority Criterion

The Majority Criterion is perhaps the simplest of the fairness criteria. It states that

"If a candidate is the first choice of more than half the voters, then the election method selects that candidate as the winner." This criterion captures the idea of majority rule that is a fundamental part of democratic choice and simply requires the voting method to produce the outcome a majority of the public selected.

The Condorcet Winner Criterion

To describe this criterion, we first have to define what a Condorcet winner is. Consider a race with three or more candidates. Suppose that to determine a winner, we ran a head-to-head election with each possible pair of candidates separately, called pairwise runoffs. After doing so, we found a candidate that won every pairwise runoff in which they were included. This means that in head-to-head matchups with every other candidate individually, they received a majority of votes. Such a candidate is called a Condorcet winner, named after Nicolas de Caritat, Marquis de Condorcet.

For example, consider a race between candidates A, B, and C. We run each possible head-to-head matchup (a.k.a. pairwise runoff) and find the results in Table 5.

Hypothetical Election 1			
Matchup (Pairwise Runoff)			Winner
A (67%)	vs.	B (33%)	A
A (55%)	vs.	C (45%)	A
B (51%)	vs.	C (49%)	B

Table 5. Pairwise runoffs for Hypothetical Election 1 resulting from a race between three candidates A, B, and C. A is the Condorcet winner and C is the Condorcet loser.

In this case, A wins both pairwise runoffs that they are involved in and hence, A is the Condorcet winner.

It may seem somewhat impractical to check for a Condorcet winner, especially since the description above suggested running three separate races that voters had to vote in. However, the Condorcet winner can easily be read off ranked choice data. Suppose in the election above, instead of running separate pairwise runoffs, we simply had each voter complete a ranked choice ballot. The ballots might have resulted in Table 6, which indicates how many voters ranked candidates in each order. Such a table is called a preference profile.

Hypothetical Election 1						
# of Votes	20%	10%	25%	6%	37%	2%
Order	A B C	A C B	B A C	B C A	C A B	C B A

Table 6. Preference profile of Hypothetical Election 1 resulting from ranked-choice ballots in a race between three candidates A, B, and C. This table results in the pairwise runoffs in Table 5.

For any preference profile, we can simulate each pairwise runoff by eliminating all the candidates not involved in the desired runoff. In the example in Table 6, eliminating candidate C from every column and adding up all the A>B columns and B>A columns yields the A vs. B pairwise runoff in Table 5. Eliminating candidate B yields the A vs. C runoff. And eliminating candidate A yields the B vs. C runoff. Thus, a Condorcet winner, if there is one, can always be detected in any voting method that uses a ranked choice ballot as its voter opinion data collection method.

Not every race of three or more candidates will necessarily have a Condorcet winner, but if there is one, it seems reasonable that that candidate ought to be declared the winner. This leads us to the Condorcet Winner Criterion, which states that

"If there is a Condorcet winner, then the election method selects that candidate as the winner."

The Equal Vote Coalition claims that methods that satisfy this property are regarded, almost universally, as the most fair and representative ways to interpret a ranked choice ballot (Equal Vote Coalition, 2024).

The Condorcet Loser Criterion

The concept of a Condorcet loser is similar to a Condorcet winner. A Condorcet loser is a candidate that, when every possible pairwise runoff is run, loses in every pairwise runoff in which they are involved. That is, the Condorcet loser loses to every other candidate in a head-to-head matchup. In Table 5, candidate C is an example of a Condorcet loser because they lose to both B and A. Similarly to a Condorcet winner, not every election has a Condorcet loser.

In the same sense that a Condorcet winner is a consensus winner, a Condorcet loser would be a consensus loser and hence, should not win the election. As such, the Condorcet Loser Criterion states that

"If there is a Condorcet loser, then the election method will NOT select that candidate as the winner."

The Clone Invariance Criterion

This is the idea of avoiding a traditional spoiler candidate. A spoiler candidate is typically considered a candidate in a single-choice plurality election that is similar to one of the leading candidates. This similarity draws away votes from that leading candidate until that candidate no longer wins. For example, Ross Perot was a potential spoiler candidate for both George H.W. Bush in the 1992 election and Bob Dole in the 1996 election. The possibility of spoiler candidates is considered problematic and hence this criterion states that

"Any candidate's ability to win is not harmed nor benefitted by the existence of an identical/similar candidate (political position-wise) as themselves in the election."

To be specific, the meaning of "identical/similar" in this criterion is that every voter ranks these candidates next to each other in their rankings with no other candidates in between. This is a good property to satisfy and eliminates the possibility of a more traditional spoiler candidate, but this is not the only way in which a candidate can spoil an election.

The Independence of Irrelevant Alternatives Criterion

This criterion expands the notion of a spoiler candidate to any candidate, similar or not, that, by their presence in the election, affects the overall ranking of other candidates. The general idea of this criterion is that we do not want the overall collective ranking of two candidates to be affected by whether a third unrelated candidate runs in the election.

To be more precise, the statement of this criterion is

"If the election method determines that candidate A is preferred to candidate B when candidate X is not running, then if candidate X instead runs for the election, then the method will not determine that candidate B is now preferred to candidate A."

In other words, the election method's calculation of the preference between candidates A and B is never affected by candidate X's presence in the election. This would guard against any possible notion of a candidate spoiling an election for another candidate, but is much more complicated to satisfy than simple clone invariance.

The Monotonicity Criterion

The idea behind this criterion is simple: if a candidate increases support for themselves, it should only improve their chances of winning the election. This seems clearly desirable and so this fairness criterion states that it should always happen. Specifically, this criterion states

"No candidate can harm their ability to win by increasing their support."

It may seem odd at first that this issue is something one would need to worry about, but not every election method satisfies this criterion, and so it becomes relevant to our discussion.

The No Dictators Criterion

This criterion is another fairly simple one. It states

"No single voter has the power to completely determine the winner."

This is a clearly desirable and an easily satisfied criterion in practice in society. It is more relevant when discussing voting in the context of a board of directors or a committee. We mention it here since it is relevant in discussing a few important theorems about voting theory below.

The Pareto Efficiency Criterion

This criterion is adapted from an idea in economics named after Vilfredo Pareto. Applied to voting methods, it is the idea that if the electorate has a unanimous preference, then the voting method cannot indicate the opposite preference. More precisely, it is the following requirement on a voting method.

"If every voter prefers candidate A to candidate B, then the election method cannot determine that candidate B is collectively preferred to candidate A."

This is a clearly desirable property and is satisfied by practically all voting methods, including the ones we discuss in this report.

The Unanimity Criterion

A related criterion to Pareto Efficiency is the idea of unanimity. It says simply

"If every voter ranks candidate A first, then the election method selects A as the winner."

Again, this is a clearly desirable property and is satisfied by all the election methods we will discuss in this report. It implies the majority condition as well. We mention it here because like the No Dictators Criterion, it is relevant in discussing a few important theorems about voting below.

The Strategy-Proof Criterion

This final criterion attempts to collect together all the different types of strategy that voters might use to take advantage of a voting method failing a fairness criterion, and as such, can almost be thought of as the conglomeration of all other fairness criteria. It is also a direct statement of one of the goals that we set out when discussing the types of behavior voting methods should encourage. This criterion is the following statement and is applied directly to either a rank-order or single-choice ballot.

"Every voter obtains the best result by ranking candidates according to their honest ranking, that is, no voter can obtain a result they prefer more by voting strategically."

This is exactly the idea that we want a voting system to incentivize honest voting and disincentivize strategic voting.

Voting Methods and Fairness Criteria

The above fairness criteria are just a small sample of the wide array of fairness criteria that have been considered. A great deal of research has been done to determine which kinds of voting methods satisfy these criteria. A recent paper by Aazami and Bray (Aazami & Bray, 2023) considered the Majority, Condorcet Winner, Clone Invariance, and Monotonicity Criteria as well as another criterion called the Last Place Loser Independence Criterion. They classified seventeen different voting methods and determined which ones satisfied these fairness criteria. They found that the only well-known voting method using ranked choice ballots that is known to satisfy all five of these criteria is Ranked Pairs (RP).

The Wikipedia page "Comparison of Electoral Systems" also has an excellent table that collects nearly two dozen voting methods and over a dozen fairness criteria and indicates which methods satisfy which fairness criteria (Wikipedia, 2024). The table also indicates which kind of ballot each method uses; over half use a ranked choice ballot.

For the sake of comparison, and because the Navratil and Smith paper references SV, we describe next the methods of RP and SV.

The Ranked Pairs Method

RP is a more modern form of ranked choice voting that was first proposed by Nicolaus Tideman in 1987. We mention this method here to describe an alternative to IRV that uses a ranked choice ballot. To best respond to some of the concerns made about IRV, we need to be able to isolate the difference between a concern about the voter opinion data collection method (i.e., the ballot type) and the voter opinion interpretation method. To do that, it will help to have two different methods that both use a ranked choice ballot.

RP is designed around the idea of a Condorcet winner, which we described above. We mentioned that there is not always a Condorcet winner in a ranked-choice election. This is because it is possible in the pairwise runoffs for the electorate to prefer candidate A to candidate B, to prefer candidate B to candidate C, and to prefer candidate C to candidate A, creating a sort of "rock, paper, scissors" issue. This phenomenon is called a Condorcet cycle. RP resolves Condorcet cycles by prioritizing the pairwise runoffs according to strength of victory.

That is, RP does the following process:

1. Computes every pairwise runoff (obtained by using a ranked-choice ballot as described in the section on the Condorcet Winner Criterion).
2. Ranks each pairwise runoff according to strength of victory, that is, from largest margin of victory to smallest.

3. Locks in each electorate preference gained from each pairwise runoff from the largest victory to smallest victory, including anything that preference and previously locked in preferences imply transitively about the electorate's overall preferences.
4. Anytime we encounter a preference that contradicts a previously locked in preference, we reject that preference because it has a weaker claim to it than the locked in preferences that are based on larger victories. This is how RP resolves the Condorcet cycles—by rejecting the weakest claim of preference in each one.
5. This is perhaps better described via an example. Consider an election of four candidates that used a ranked choice ballot resulting in the preference profile in Table 7. As described before, we simulate each pairwise runoff by eliminating all but the two candidates we want to compare and looking at how many prefer one of the candidates to the other. That is, for the runoff of A and B, we look at the table that results from eliminating C and D and combine the columns that have A preferred to B. Using RP, we will list these in order of margin of victory from largest margin to smallest. The result is in Table 8.

Hypothetical Election 3								
# of Votes	15%	25%	5%	4%	16%	5%	5%	25%
Order	A B C D	A B D C	A C D B	B C A D	B C D A	C D A B	D B C A	D C A B

Table 7. Preference profile of Hypothetical Election 3 resulting from ranked choice ballots in an election with four candidates. This election contains two Condorcet cycles: A>B>C>A and A>B>D>A.

Hypothetical Election 3				
Matchup (Pairwise Runoff)			Winner	Margin
A (75%)	vs.	B (25%)	A	50%
B (65%)	vs.	C (35%)	B	30%
B (60%)	vs.	D (40%)	B	20%
C (55%)	vs.	A (45%)	C	10%
D (55%)	vs.	C (45%)	D	10%
D (51%)	vs.	A (49%)	D	2%

Table 8. Pairwise runoffs for Hypothetical Election 3 resulting from a race between four candidates A, B, C, and D. This election contains two Condorcet cycles: A>B>C>A and A>B>D>A.

From Table 8, we see that A is preferred to B and B is preferred to C. This should suggest that the electorate prefers A to C as well, but it turns out that C is actually preferred to A. Hence, we have a Condorcet cycle A>B>C>A. Similarly, A is preferred to B, B is preferred to D, but D is preferred to A. This creates another Condorcet cycle A>B>D>A. Since every candidate is involved in a Condorcet cycle, this election has neither a Condorcet winner nor a Condorcet loser. To resolve this, RP prioritizes stronger victories.

Note that the victory of A over B is quite large (the margin is 50%), while those of C and D over A are quite small by comparison. The process of RP will lock in the preference A over B from the first victory on the list. That is, we lock in that A > B.

The second largest victory locks in the preference of B over C. Since both the preferences "A preferred to B" and "B preferred to C" are locked in, we also lock in the transitively implied preference "A preferred to C." This yields the preference order A > B > C.

The third strongest victory locks in the preference of B over D. Again, since both the preferences "A preferred to B" and "B preferred to D" are locked in, we also lock in the implied preference "A preferred to D." This yields the preference order A > B > D.

The next largest victory is C over A, but this preference contradicts the previously locked in preference of A over C, so we eliminate this preference from consideration because it has a weaker claim to it than the locked in preference.

The second to last victory is the preference of D over C. This does not contradict any previously locked in preference and, as such, we lock in the preference D > C. Combining this with the previously locked in preferences of A > B > C and A > B > D, we have the overall ranking A > B > D > C.

The final victory is the smallest with a margin of only 2%. This victory has D over A, but that contradicts a previously locked in victory of A over D and hence, we eliminate this preference.

The result of RP on this election is the ordering A > B > D > C. This declares A the winner in a single-winner election.

The Score (Range) Voting Method

In this report we responded to several concerns brought by a paper written by Navratil and Smith analyzing IRV results in Utah during the 2021 election cycle (Navratil & Smith, 2022). To understand the motivation that Navratil and Smith have to criticize IRV, one must note that their purpose is to promote their favored election method, Score Voting (SV), which is also referred to as Range Voting. To help make our discussion of their paper clearer, we provide a brief explanation of SV here.

The idea behind SV is that we should collect even more data about a voter's opinion of each candidate than their relative ranking of each candidate. To accomplish this, SV uses, as a voter opinion data collection method, a scoring ballot where voters are asked to score each candidate individually on a scale of, for instance, one to nine—though the scale could be larger if desired. (On rangevoting.org, they initially recommend scoring 0–99, but point out that mathematically it does not really matter (Smith, 2005).) Voters are also allowed to indicate that they have no opinion and provide a score of "X" to indicate that.

To interpret score ballots of this type, SV simply identifies the candidate with the highest average score and declares that candidate the winner.

Two Most Unfortunate Mathematical Facts About Voting

With this discussion about mathematically analyzing voting systems, one might conclude that we should simply appeal to mathematics and let it determine what is the best method. Or to use mathematics to identify a method that satisfies all the fairness criteria so that it accomplishes the purpose of voting we set out for, and incentivizes the right kinds of behavior.

Unfortunately, the mathematics of this situation is far more complicated than that. Kenneth Arrow proved the following theorem in 1950 that showed that no voting method that could use a ranked choice ballot, which includes plurality, IRV, and RP, which satisfies No Dictators and Pareto Efficiency, is completely immune to the spoiler effect (Arrow, 1950). This is delightfully called Arrow's Impossibility Theorem. The more modern version of this theorem is the following.

Arrow's Impossibility Theorem

No rank-order voting system for three or more candidates that has no dictator can satisfy both the fairness criteria Pareto Efficiency and Independence of Irrelevant Alternatives.

Arrow's Impossibility Theorem is interesting because the criteria of No Dictators and Pareto Efficiency are both simple criteria to satisfy and tend to be satisfied by most of the popular voting methods. Thus, this theorem shows that the spoiler effect is very difficult to avoid, partly because there are many ways that a candidate can be considered a spoiler. We note here that Arrow's Impossibility Theorem does not say that some kinds of spoilers cannot be avoided and indeed some ranked choice voting methods are immune to specific types of spoilers.

Arrow's Impossibility Theorem only applies to voting methods that can be computed from a ranked choice ballot. Plurality is included because it can be done with either a ranked choice ballot or a single-choice ballot. However, the theorem does not apply to SV, which uses a score ballot. This should not be taken to mean that SV, which satisfies the No Dictators and Pareto Efficiency criteria, automatically avoids the spoiler effect. There is more that would need to be shown to make that claim.

The Gibbard-Satterthwaite Theorem also raises an unfortunate fact about voting. This theorem was proved independently by Allan Gibbard in 1973 (Gibbard, 1973) and Mark Satterthwaite in 1975 (Satterthwaite, 1975). This theorem shows that in ranked choice voting systems, the only system that is not subject to manipulation by strategic voting is a dictatorship. Specifically, the theorem states the following.

The Gibbard-Satterthwaite Theorem

Suppose we have a ranked choice voting method in a race of three or more candidates. Suppose also that each voter is required to have a preference list of all candidates with no ties allowed. If the method satisfies the Unanimity, Pareto Efficiency, and Strategy-Proof criteria, then the method is a dictatorship.

Recall that the Unanimity, Pareto Efficiency, and No Dictators criteria are almost always satisfied by the voting methods we describe here. Then this theorem states that all such voting methods are not strategy-proof. Thus, this theorem really says that no reasonable ranked choice voting method is completely immune to manipulation by strategic voting. This theorem is then disappointing in the sense that it means that no ranked choice voting method, including plurality, can perfectly satisfy our stated purpose and goal of voting. One might think to move away from ranked choice voting methods because the theorem says nothing about other types and so maybe there is hope. However, other systems, like SV, are still typically susceptible to strategic voting and so the answer is not that simple. Instead, we should look for methods that get us closest to the purpose and goal of voting.

This then suggests that we might compare voting methods by which fairness criteria they satisfy. We discussed this for the four methods presented here in the section “Fairness Criteria and Four Voting Methods.”

We mentioned in that section that SV fails both Condorcet Criteria and the Independence of Irrelevant Alternatives Criterion. We conclude this Appendix with two example elections that illustrate these failures. To see how SV fails both Condorcet criteria, we provide an example of an election where SV results in not only failing to elect a Condorcet Winner, but in fact, elects a Condorcet Loser. Such an election is possible if we assume that all voters provide an honest appraisal of each candidate that is independent of which candidates are running. Following that, we provide an example election where SV fails the Independence of Irrelevant Alternatives Criterion.

An Example Election Where Score Voting Fails Both Condorcet Criteria

Consider an election between three candidates A, B, and C. Suppose that A and B belong to a party that has a strong two-thirds majority support in an electorate, while C belongs to a party with minority support. C supporters are very enthusiastic about their candidate and are not particularly keen about the candidates from the major party, but if pressed would pick A over B practically all the time. On the other hand, the major party’s candidates are mediocre even to the party. Their supporters are conflicted over which one is better. They all prefer both candidates to C by far, but are not particularly enthused about either A or B.

At the ballot box, this electorate is instructed to use SV and score the three candidates on a scale from one to nine on how much they like the candidate. For simplicity, suppose that 3000 voters voted as in Table 9.

Candidate / # of Votes	1000	1000	1000	Average
A	5	3	2	3.33
B	3	5	1	3.00
C	1	1	9	3.67

Table 9. Hypothetical SV election that shows that SV fails both Condorcet criteria. Candidate A is the Condorcet winner, but SV elects the Condorcet loser, C.

Since C has the highest average score, C is declared the winner. However, if we consider each pairwise runoff, we find that between A and B, all those that scored C with 9 prefer A to B, and so A would win this pairwise runoff 2000 votes to 1000 votes. Between A and C, all those that scored B highest prefer A to C, and hence A wins this pairwise runoff again 2000 votes to 1000 votes. Between B and C, all those that scored A highest prefer B to C, and hence B wins this pairwise runoff 2000 votes to 1000 votes.

Since A wins both pairwise runoffs they’re considered in, A is the Condorcet winner. SV failed to elect them in this example, thus SV fails the Condorcet Winner Criterion. Moreover, C lost both pairwise runoffs they’re considered in, and hence C is the Condorcet loser. However, SV elected C in this example. Thus, SV also fails the Condorcet Loser Criterion. The reason why this can happen is due to the enthusiastic support for the minority candidate compared to apathetic support for two majority candidates.

Two other major problems of SV are illustrated here as well. First, we see the issue that enthusiastic support is counted more heavily than less enthusiastic support. Each C voter’s nine was counted nearly as much as two of A voters' score of five. That’s the only reason why a candidate with so little support was able to stay competitive in a race like this.

Second, it seems foolish for A or B voters to only score their top candidate a five when that lets candidate C win. But if we assume here that voters actually voted for their honest level of approval of each of the candidates, things like this can happen. The real problem is that everyone in the electorate will interpret how to score their candidates differently. Some might take it as an honest appraisal of the candidates, in which case they might not rank any candidate maximum because there’s no such thing as a perfect candidate. Others might be more strategic and rank candidates higher than their actual approval level in order to maximize their chances of a better outcome. The naïve assumption that many proponents of SV make is that voters will act the way that they think is optimal. If the public experiment with IRV has taught us anything, it is that it is very difficult for the electorate to identify the strategy that is optimal, and hence they make decisions that are mathematically bad unintentionally, such as not filling out a complete ranking. The reality of SV is going to have similar issues, and some voters will vote their mind while others will vote strategically. Because of this, there is no reason to conclude that, in practice, SV would always elect a Condorcet winner or always avoid electing a Condorcet loser.

Ultimately though, SV seems to encourage strategic voting and appears to be one of the voting methods most susceptible to it because it is the voting method that has the widest range of options for voters to choose from.

An Example Election Where Score Voting Falls Victim to a Spoiler Candidate

One claim about SV is that it satisfies the Independence of Irrelevant Alternatives Criterion. However, this is only true if every voter scores candidates on a scale that is independent of which candidates are running. In the previous section, we suggested that such a strategy might lead to voters not giving the maximum score to any candidate and produce bad outcomes. In this section, we discuss a hypothetical election where we consider an electorate that makes the more strategic choice of always scoring their favorite candidate with the maximum score and giving honest appraisals of the remaining candidates.

Suppose we have another election of three candidates A, B, and C. This time the electorate scores them from one to nine as in Table 10. Again, for simplicity, we suppose that 3000 voters voted and that they were split three ways in how they scored the candidates.

Candidate / # of Votes	1000	1000	1000	Average
A	9	1	7	5.67
B	1	9	1	3.67
C	3	4	9	5.33

Table 10. Hypothetical SV election that shows that SV fails Independence of Irrelevant Alternatives. Assuming that each voter will rank their top candidate 9 and all other candidates honestly, then in the election above, Candidate B acts as a spoiler for Candidate C.

In this election, SV elects Candidate A. However, suppose that Candidate B knew they would not win the election and so drops out before the election. Since all the voters in this hypothetical electorate decided to score their top candidate with the maximum score and all the other candidates honestly, when Candidate B drops out, the 1000 voters that scored B with a 9, now will change their rankings of C to a 9 and leave their ranking of A the same. This produces the election in Table 11.

Candidate / # of Votes	1000	1000	1000	Average
A	9	1	7	5.67
C	3	9	9	7.00

Table 11. Part 2 of a hypothetical SV election that shows that SV fails Independence of Irrelevant Alternatives. This table shows the voting after Candidate B dropped out of the election in Table 6. Assuming that each voter will rank their top candidate 9 and all other candidates honestly, while Candidate A won the election when Candidate B was present, Candidate C wins this one. Thus, Candidate B acted as a spoiler for Candidate C.

Candidate C now handily wins the election via SV. This means that Candidate B acted as a spoiler in this election for Candidate C. Thus, if we assume that all voters will score their favorite candidate maximum, we have scenarios where SV is not immune to the spoiler effect. Since in reality at least some voters will indeed use this strategy, there is no reason to conclude that, in practice, SV would not be susceptible to a spoiler effect. Since SV, in practice, fails both Condorcet criteria, is susceptible to spoilers, and seems incredibly prone to strategic voting, it seems hardly better than plurality.

At any rate, the situation is complicated. The two examples presented here made two different assumptions about voter behavior. The first example showed that assuming that voters will always submit a ranking that is their honest appraisal of candidates and hence independent of which candidates run may lead to violating both Condorcet criteria. On the other hand, assuming that voters always rank some candidate maximum may lead to violating Independent of Irrelevant Alternatives. Since SV seems to require contradictory assumptions to satisfy the Condorcet and Independence of Irrelevant Alternatives criteria, we indicated in the section "Fairness Criteria and Four Voting Methods" that they fail these criteria. Moreover, neither assumption (either giving an independent appraisal or always scoring their favorite candidate highest) is realistic since the electorate will more likely do a combination of the two. Thus, it is likely that SV fails all these criteria in practice.

References

108th Congress of the United State of America. (2003). Our American Goverment. Washington D.C.: U.S. Goverment Printing Office.

Aazami, A. B., & Bray, H. L. (2023). Classification of Preferential Voting Methods. Constitutional Political Economy, 34, 510-523. doi:10.1007/s10602-022-09384-8

Anderson, J. (2023, October 25). Explaining New NBA In-season Tournament, Flop Rule, Rest Rule, Expanded Coach's Challenge. The Sacramento Bee. Retrieved from https://www.sacbee.com/sports/nba/sacramento-kings/article280918808.html

Arrow, K. J. (1950, August). A Difficulty in the Concept of Social Welfare. Journal of Political Economy, 58(4), 328-346.

Black, D. (1948, Feb). On the Rationale of Group Decision-Making. Journal of Political Economy, 56(1).

Davidson, A. (2024). Position on Issues - Ranked Choice Voting. Retrieved August 26, 2024, from davidsonclerk.com: https://davidsonclerk.com/#tve-jump-1704475b081

Duverger, M. (1954). Political Parties: Their Organization and Activity in the Modern State. Methuen.

Eggers, A. C., & Nowacki, T. (2024, April). Susceptibility to Strategic Voting: A Comparison of Plurality and Instant-Runoff Elections. The Journal of Politics, 86(2). doi:https://doi.org/10.1086/726943

Equal Vote Coalition. (2024). Condorcet Voting. Retrieved August 23, 2024, from equal.vote: https://www.equal.vote/minimax

Gibbard, A. (1973). Manipulation of Voting Schemes: A General Result. Econometrica, 41(4), 587-601.

Gofton, W. T., Papp, S. R., Gofton, T., & Beaulé, P. E. (2016). Understanding and Taking Control of Surgical Learning Curves. Instructional Course Lectures, 65, 623-631. Retrieved from https://pubmed.ncbi.nlm.nih.gov/27049228/

Hutchinson, R., & Parry, A. (2023). What if Voters Don't Rank All the Candidates: Inactive Ballots in Single-Choice vs. Instant Runoff Voting. Orem: Gary R Herbert Institute for Public Policy and FairVote. Retrieved from https://fairvote.org/report/what-if-voters-dont-rank-all-the-candidates-exhausted-ballots-in-single-choice-vs-instant-runoff-voting/

Kimball, D. C., & Anthony, J. (2016). Voter Participation with Ranked Choice Voting in the United States. Retrieved from https://www.umsl.edu/~kimballd/KimballRCV.pdf

Maloy, J. (2020). Voting Error across Multiple Ballot Types: Results from Super Tuesday (2020) Experiments in Four American States. Social Science Research Network. doi:https://dx.doi.org/10.2139/ssrn.3697637

Maloy, J. S., & Ward, M. (2021). The Impact of Input Rules and Ballot Options on Voting Error: An Experimental Analysis. Politics and Governance, 9(2). doi:https://doi.org/10.17645/pag.v9i2.3938

Mohsin, F., Han, Q., Ruan, S., Chen, P.-Y., Rossi, F., & Xia, L. (2024). Computational Complexity of Verifying the Group No-show Paradox. Proceedings of the Thirty-Third International Joint Conference on Artificial Intelligence (pp. 2958-2966). Jeju: International Joint Conferences on Artificial Intelligence.

Moulin, H. (1988, June). Condorcet's Principle Implies the No Show Paradox. Journal of Economic Theory, 45(1), 53-64.

Navratil, J., & Smith, W. D. (2022). Analysis of the 2021 Instant Run-Off Elections in Utah. viXra.org. Retrieved from https://vixra.org/abs/2208.0166

Neely, F., & McDaniel, J. (2015). Overvoting and the Equality of Voice under Instant-Runoff Voting in San Francisco. California Journal of Politics and Policy, 7(4). doi:https://doi.org/10.5070/P2cjpp7428929

Office of the Lieutenant Governor. (2020). 2020 Regular Primary Election Canvass. Salt Lake City, UT: State of Utah. Retrieved from https://voteinfo.utah.gov/wp-content/uploads/sites/42/2020/07/2020-Primary-Election-Canvass.pdf

Palfrey, T. R. (1989). A Mathematical Proof of Duverger's Law. In Models of Strategic Choice in Politics (pp. 69-91). Ann Arbor: University of Michigan Press.

Parry, A., & Kidd, J. (2024). Deficiencies in Recent Research on Ranked Choice Voting Ballot Error Rates. Wellesley, MA: Institute for Mathematics and Democracy.

Pettigrew, S., & Radley, D. (2023). Ballot Marking Errors in Ranked-Choice Voting. Social Science Research Network. doi:https://dx.doi.org/10.2139/ssrn.4670677

Satterthwaite, M. A. (1975). Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions. Journal of Economic Theory, 10(2), 187-217.

Smith, W. D. (2005). Retrieved August 24, 2024, from rangevoting.org: https://rangevoting.org/RangeVoting.html

USA Basketball. (2013, December 31). History of the 3-Pointer. Retrieved from USAB.com: https://www.usab.com/news/2014/01/history-of-the-3-pointer

Utah Code 20A-4-6. (2018, May 8). Municipal Alternative Voting Methods Pilot Project. Retrieved from https://le.utah.gov/xcode/Title20A/Chapter4/20A-4-P6.html

Utah State Legislature 2018 General Session. (2018, March). H.B. 35 Municipal Alternate Voting Methods Pilot Project. Retrieved from Utah State Legislature: https://le.utah.gov/~2018/bills/static/HB0035.html

Wallis, W. D. (2014). The Mathematics of Elections and Voting. Springer.

Watson, J. (2013). Strategy: An Introduction to Game Theory. New York: W. W. Norton & Company.

Wikipedia. (2024). Comparison of Electoral Systems. Retrieved August 23, 2024, from wikipedia.org: https://en.wikipedia.org/wiki/Comparison_of_electoral_systems

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