Solving Systems of Linear Equations

UVU Math Lab

Method of Substitution:

We use substitution usually when it is easy to solve one of the equations for x or y because it has a coefficient of 1.

Solve the given system of equations:

$$\begin{cases} 2x + 3y = 3 \\ 3x - 4y = 13 \end{cases}$$

STEP ONE: We number each equation.	(1) $2x + 3y = 3$ (2) $3x - 4y = 13$
STEP TWO: Choose one equation and solve for one variable.	(1) $2x + 3y = 3$ $2x = 3 - 3y$ $x = \frac{3}{2} - \frac{3}{2}y$
STEP THREE: Substitute the expression found in step 1 for the variable in the second equation to get an equation in one variable.	(2) $3\left(\frac{3}{2} - \frac{3}{2}y\right) - 4y = 13$ $\frac{9}{2} - \frac{9}{2}y - \frac{8}{2}y = \frac{26}{2}$ $\frac{-17}{2}y = \frac{17}{2}$ $y = -1$
STEP FOUR: Back substitute the solution for the variable you have solved for into either one of the original 2 equations and solve for the other variable.	(1) $2x + 3(-1) = 3$ $x = 3$
STEP FIVE: Write your solution as an ordered pair.	$(3, -1)$

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Method of Elimination:

Solve the given system of equations:

$$\begin{cases} 2x + y + 2 = 11 \\ 3x + 2y + 2z = 8 \\ x + 4y + 3z = 0 \end{cases}$$

STEP ONE: Number each equation.	(1) $2x + y + 2 = 11$ (2) $3x + 2y + 2z = 8$ (3) $x + 4y + 3z = 0$
STEP TWO: Choose any two equations and eliminate one variable using the elimination method. We add equations (1) and (3) and eliminate the x variable by multiplying equation (3) by – 2. The new equation is equation (4)	(1) $2x + y + 2 = 11$ $-2(3)$ $\underline{-2x - 8y - 6z = 0}$ (4) $-7y - 4z = 11$
STEP THREE: Choose a DIFFERENT pair of equations and eliminate the SAME variable. We will add equations (2) and (3) and eliminate the x variable by multiplying equation (3) by – 3. The new equation is (5).	(2) $3x + 2y + 2z = 8$ $-3(3)$ $\underline{-3x - 12y - 9z = 0}$ (5) $-10y - 7z = 8$
STEP FOUR: Equations (4) and (5) create a system of two equations in two variables which we already know how to solve.	(4) $-7y - 4z = 11$ (5) $-10y - 7z = 8$ ↓ $7(4)$ $-49y - 28z = 77$ $-4(5)$ $\underline{40y + 28z = -32}$ $-9y = 45$ $y = -5$
STEP FIVE: Back substitute the solutions for the 2 variables you have solved for into any one of the original 3 equations and solve for the third variable. Likewise, we will substitute $y = -5$ and $z = 6$ into equation (3).	(4) $-7(-5) - 4z = 11$ $-4z = -24$ $z = 6$ (3) $x + 4(-5) + 3(6) = 0$ $x = 2$
STEP SIX: Write your solution as an ordered triple:	(2, -5, 6)

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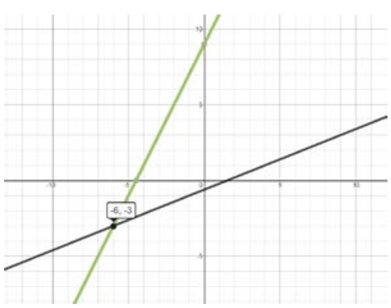
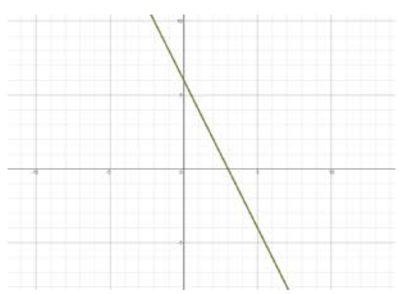
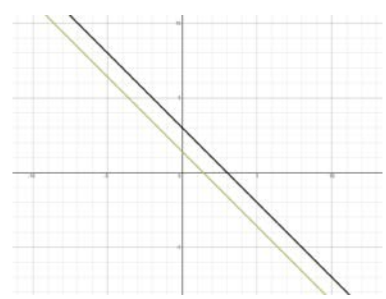
Systems of Linear Equations in Two Variables:

Given	$\begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases}$	$\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases}$	$\begin{cases} x + y = 3 \\ 2x + 2y = 7 \end{cases}$
Solve Algebraically	$y - 2x = 9$ $y = 2x + 9$ $2x - 5(y) = 3$ $2x - 5(2x + 9) = 3$ $2x - 10x - 45 = 3$ $-8x = 48$ $x = -6$ $y = 2(-6) + 9$ $y = -3$ $(-6, -3)$	$4x + 2y = 12$ $\underline{2 \cdot (-2x - y = -6)}$ $4x + 2y = 12$ $\underline{-4x - 2y = -12}$ $0 = 0$ These equations will have the same slope and the same y-intercept. They are the *same line*!	$x + y = 3$ $y = 3 - x$ $2x + (y) = 7$ $2x + 2(3 - x) = 7$ $2x + 6 - 2x = 7$ $6 \neq 7$ These equations will have the same slope and different y-intercepts. They are *parallel lines*.
Solve Graphically	$\begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases} \rightarrow \begin{cases} y = \frac{2}{5}x - \frac{3}{5} \\ y = 2x + 9 \end{cases}$ 	$\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases} \rightarrow \begin{cases} y = -2x + 6 \\ y = -2x + 6 \end{cases}$ 	$\begin{cases} x + y = 3 \\ 2x + 2y = 7 \end{cases} \rightarrow \begin{cases} y = -x + 3 \\ y = -x + \frac{7}{2} \end{cases}$ 
Then we say…	Consistent System Independent Equations One Unique Solution	Consistent System Dependent Equations Infinitely Many Solutions	Inconsistent System No Solution