Delta Epsilon Proofs

Example # 1

For $\lim_{x \to 2} 3x - 1 = 5$ find an $\varepsilon > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$

Step 1: Find a suitable δ

$|(3x-1)-5| < \varepsilon$ => $|3x-6| < \varepsilon$

$3|x-2| < \varepsilon$ => $|x-2| < \frac{\varepsilon}{3}$

$\therefore$ it is reasonable to (suitably) pick $\delta = \frac{\varepsilon}{3}$

Step 2: Proof. Since $\delta = \frac{\varepsilon}{3}$ then $|x-2| < \frac{\varepsilon}{3}$

$3|x-2| < \varepsilon$ => $|3x-6| < \varepsilon$

$|(3x-1)-5| < \varepsilon$ => $|f(x) - L| < \varepsilon$

$\therefore$ true (Note: if f(x) = mx+b then $\delta = \frac{\varepsilon}{m}$)

Example # 2

For $\lim_{x \to 2} 3x^2 - 4x + 1 = 5$ find an $\varepsilon > 0$ such that if $0 < |x-2| < \delta$ then $|(3x^2 - 4x + 1) - 5| < \varepsilon$

Step 1: Find a suitable δ

A) Factor $3x^2 - 4x - 4 = 0$ to get $(x - 2)(3x + 2)$

Suppose $\exists$ a 'c' such that $c|x-2| < \varepsilon$ and $|3x+2| < c$

Then it follows that

$|3x+2||x-2| < c|x-2|$ and $|x-2| < \frac{\varepsilon}{c}$

$|3x+2||x-2| < c|x-2| < \varepsilon$

$|3x+2||x-2| < \varepsilon$ only if $|x-2| < \frac{\varepsilon}{c}$

$\therefore \delta = \frac{\varepsilon}{c}$

B) We need to find a value for c. Pick $\delta > 0$ say 1.
Start with the premise |x - 2| < 1 then make it look like |3x + 2| < c.

$|x - 2| < 1$

$0 < x - 2 < 1$ (add 2)

$2 < x < 3$ (multiply by 3)

$6 < 3x < 9$ (add 2)

$-11 < 8 < 3x + 2 < 11$

$-11 < 3x + 2 < 11$

$|3x + 2| < 11$ $\therefore$ c = 11 $\therefore \delta = \min(1, \frac{\varepsilon}{11})$

Step 2: Proof.

Part A becomes the proof and $|3x+2| < 11$

Is no longer an assumption since it was derived.

Pick $\delta = \frac{\varepsilon}{11}$ then it follows that:

$|3x+2| < 11$ and $|x-2| < \frac{\varepsilon}{11}$

$|x-2||3x+2| < 11|x-2|$ and $11|x-2| < \varepsilon$

$|x-2||3x+2| < 11|x-2| < \varepsilon$

$|x-2||3x+2| < \varepsilon$ so $|(3x^2 - 4x + 1) - 5| < \varepsilon$

> Note that "c" is no longer a constant, but a linear function $\therefore |3x+2| < c$. Or in other words, you find a "c" so that you have $c|x-a|$, thus $|3x+2||x-2|$.