Tabular Integration

Tabular Integration can be used as a viable substitute for Integration by Parts when you have a terminating u term.

Take the following as an example: $\int x^3 e^{2x} dx$. When doing integration by parts $u = x^3$ and $dv = e^{2x}dx$

We will use these two terms in our tabular integration

Differentiate ↓	Integrate ↓
x^3	e^{2x}
$3x^2$	$\frac{1}{2} e^{2x}$
$6x$	$\frac{1}{4} e^{2x}$
6	$\frac{1}{8} e^{2x}$
0	$\frac{1}{16} e^{2x}$

Now, multiply down the diagonals alternating signs, starting with + (then –). For example…

$$x^3 \frac{1}{2} e^{2x} - 3x^2 \frac{1}{4} e^{2x} + 6x \frac{1}{8} e^{2x} - 6 \frac{1}{16} e^{2x} + C$$

$$= \frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{8} e^{2x} + C$$

Check: Using Integration by Parts

$$\int x^3 e^{2x} dx \quad u = x^3 \text{ and } dv = e^{2x}dx \text{ so } du = 3x^2 dx \text{ and } v = \frac{1}{2} e^{2x}$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{2} \int x^2 e^{2x} dx \quad u = x^2 \text{ and } dv = e^{2x}dx \text{ so } du = 2x \; dx \text{ and } v = \frac{1}{2} e^{2x}$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{2} [\frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx] =$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{2} \int x e^{2x} dx \quad u = x \text{ and } dv = e^{2x}dx \text{ so } du = dx \text{ and } v = \frac{1}{2} e^{2x}$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{2} [\frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx] =$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{4} \int e^{2x} dx = \frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{4} [\frac{1}{2} e^{2x}] + C =$$

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{8} e^{2x} + C$$

Our result with tabular integration above matches the result with integration by parts.