

Tabular Integration

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This handout goes over a more efficient way of doing integration by parts called Tabular Integration.

When to Use Tabular Integration

Integration by parts is defined by the following formula:

$$\int u \, dv = uv - \int v \, du$$

Tabular integration is the most useful when integration by parts is required multiple times. In the most common uses of tabular integration, one of the elements is “cyclic”. The term “cyclic” refers to the idea that taking repeated integrals results in a loop of repeating terms. Examples include $\sin x$, $\cos x$, and e^x .

Step 1

- Start by establishing a u and dv in the initial integral.
- Select a function for u to derive to 0.
- The other function will be dv .

Step 2

- Set up a table with three columns. the first will contain alternating signs (+/-), beginning with positive.
- In the second column, take derivatives of u until you reach zero.
- In the third column, take repeated indefinite integrals of dv until you are even with the zero in the second column.

Step 3

- Take the sign and u from row one, and multiply by the dv on row two. Next, add the product of the sign and u from the second row and multiply by the dv from row three. Repeat until you hit zero in the u column.

$$\begin{aligned} & \int u \, dv \\ & \int x^2 \sin x \, dx \\ & u = x^2 \\ & dv = \sin x \, dx \end{aligned}$$

Signs	u - Derive	dv - Integrate
+	x^2	$\sin x$
-	$2x$	$-\cos x$
+	2	$-\sin x$
-	0	$\cos x$

$$\begin{aligned} & (+)(x^2)(-\cos x) \Rightarrow -x^2 \cos x \\ & (-)(2x)(-\sin x) \Rightarrow 2x \sin x \\ & (+)(2)(\cos x) \Rightarrow 2 \cos x \end{aligned}$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + c \Rightarrow 2x \sin x + (2 - x^2) \cos x + c$$