Graphing Rational Functions

UVU Math Lab

Step 1: Given a rational function, factor the numerator and denominator.

$$f(x) = \frac{2(x^2 - 2x - 3)}{x^2 + 2x}$$

$$f(x) = \frac{2(x+1)(x-3)}{x(x+2)}$$

Step 2: Identify the x-intercepts.

x-intercepts are found by setting $f(x) = y = 0$.

$$f(x) = \frac{\mathbf{2(x+1)(x-3)}}{x(x+2)} = 0$$

$$f(x) = 2(x+1)(x-3) = 0$$

$$x + 1 = 0 \qquad\qquad x - 3 = 0$$
$$x = -1 \qquad\qquad x = 3$$

Step 3: Identify the vertical asymptotes.

A vertical asymptote is an invisible line which the graph will generally not cross and is written as the equation of a vertical line, $x = a$.
The vertical asymptotes of a rational function are directly related to its domain restriction,

Given $f(x) = \frac{N(x)}{D(x)}$, we know that $D(x) \neq 0$ since the denominator of any rational expression is undefined when it is equal to zero.

$$f(x) = \frac{2(x+1)(x-3)}{\mathbf{x(x+2)}}$$

Therefore we will have vertical asymptotes at:

$$x(x+2) = 0$$

$$x = 0 \qquad\qquad x + 2 = 0$$
$$x = -2$$

Graphing Rational Functions continued:

Step 4: Make a number line table with the critical values identified above.

		V.A. -2		x-int -1		V.A. 0		x-int 3		Domain x
$x+2$	$-$	|	$+$		$+$		$+$		$+$	
$x+1$	$-$		$-$	|	$+$		$+$		$+$	
x	$-$		$-$		$-$	|	$+$		$+$	
$x-3$	$-$		$-$		$-$		$-$	|	$+$	
multiplied	$+$		$-$		$+$		$-$		$+$	Range

- Notice that the binomials are listed in the same order as their zeros, critical values.
- All values up to a binomial's zero would make that binomial a negative number. So we know the graph will be a negative y-value, that is, the graph will be below the x-axis.
- Likewise, all values after a binomial's zero would make that binomial a positive y-value and above the x-axis.
- The last row is the result of multiplying the signs in each column.

Step 5: Identify horizontal or oblique asymptotes.

Horizontal and oblique (diagonal) asymptotes are found by comparing the degree of the numerator to the degree of the denominator.

Given $f(x) = \frac{N(x)}{D(x)} = \frac{ax^n + \cdots}{bx^d + \cdots}$

If…	then the asymptote is…	and defined by…
$n < d$	horizontal	the x-axis: $$y = 0$$
$n = d$	horizontal	the ratio of the leading coefficients: $$y = \frac{a}{b}$$
$n > d$ by 1 degree	oblique (diagonal)	the equation of the line found by dividing the numerator by the denominator. $$f(x) = \frac{N(x)}{D(x)} = mx + b + \frac{r(x)}{D(x)}$$

Otherwise, there are no asymptotes.

$$f(x) = \frac{2(x^2 - 2x - 3)}{x^2 + 2x}$$

Since the degree of the numerator is equal to that of the denominator, $n = d$, then the ratio of the leading coefficients is the H.A.

$$y = \frac{2}{1} = 2$$

Graphing Rational Functions continued:

Step 6: Identify y-intercept.
The y-intercept is found by setting $x = 0$. However, since 0 is an excluded domain value, we will not have a y-intercept in this case.

Step 7: Graph.	
1. Graph asymptotes. 2. Graph x- & y-intercepts. 3. Graph curves using the table as a guide for the range values and remembering that the graph will be guided by the asymptotes.