Mathematical Induction

For a series: $a_1 + a_2 + a_3 + ... + a_n = S_n$

Consider

a_1 as the "term generator" (*nth* term) and

S_n as the sum of the first *n* terms

Mathematical induction has two parts:

1. Show that the sum of the first term equals the first term, that

$$S_1 = a_1$$

2. Show that the sum of *k* terms + the next term = the sum of *k+1* terms:

$$S_k + a_{k+1} = S_{k+1}$$

(As an example, if you added up 4 terms, then added in the next term, your sum would equal the sum of the first 5 terms.)

Example: Show that $2 + 5 + 8 + ... + (3n - 1) = \frac{n(3n+1)}{2}$ is true for all natural numbers *n.*

Note that $a_n = (3n - 1)$ and $S_n = \frac{n(3n+1)}{2}$

1. Show $S_1 = a_1$ Let *n* = 1

$$[3(1) - 1] = \frac{1[3(1)+1)}{2}$$

2=2

True

2. Show $S_k + a_{k+1} = S_{k+1}$. Substitute *k* and *k+1* in the appropriate formulas.

$$\frac{(k)[3(k)+1]}{2} + [3(k+1)-1] = \frac{(k+1)[3(k+1)+1]}{2}$$

$$\frac{3k^2+k}{2} + [3k+2] = \frac{(k+1)[3k+4]}{2}$$

$$\frac{3k^2+7k+4}{2} = \frac{3k^2+7k+4]}{2}$$

We conclude that the statement is true for all natural numbers *n.*