Mathematical Induction

In this handout, the process of mathematical induction is shown. Mathematical induction is a method of proving that a statement P(n) is true for every natural number n through a finite proof.

Definitions

For a series: $a_1 + a_2 + a_3 + \cdots + a_n = S_n$

Consider a_1 as the "term generator" (*nth* term) and S_n as the sum of the first *n* terms.

Proving mathematical induction has two parts:

Step 1
Show that the sum of the first term equals the first term, that:

$$S_1 = a_1$$

This step is generally the simplest as long as the series begins at the first term rather than beginning at a zeroth or negative numbered term.

Step 2
Show that the sum of k terms + the next term = the sum of k+1 terms:

$$S_k + a_{k+1} = S_{k+1}$$

This step is generally trickier because you must first assume that the sum of the nth terms previous is equal to the kth term. The assumption is then used to prove the above equation. As an example, if you added up 4 terms, then added in the next term, your sum would equal the sum of the first 5 terms.

Example

Prove that $2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n+1)}{2}$ is true for all natural numbers n.

Step 1
Show that the sum of the first term S_1 equals the first term a_1, that: $S_1 = a_1$. Let n = 1. Note that $a_n = (3n - 1)$ and $S_n = \frac{n(3n+1)}{2}$

$$a_1 = S_1$$

$$[3(1) - 1] = \frac{1*[3(1) + 1]}{2}$$

$$[3 - 1] = \frac{1[4]}{2}$$

$$2 = 2$$

True

Step 2

Show that $S_k + a_{k+1} = S_{k+1}$. Substitute *k* and *k+1* in the appropriate formulas.

$$\frac{(k)[3(k)+1]}{2} + [3(k+1)-1] = \frac{(k+1)[(3(k+1)+1]}{2}$$

$$\frac{3k^2}{2} + [3k+2] = \frac{(k+1)[3k+4]}{2}$$

$$\frac{3k^2+7k+4}{2} = \frac{3k^2+7k+4}{2}$$

We conclude that the statement is true for all natural numbers n.