Exponential & Logarithmic Functions

This document will go over properties and applications of exponential and logarithmic functions.

Exponential Function

An *exponential function* is of the form

$$f(x) = b^x,$$

Where b is the base and $b > 0$ and $b \neq 1$.

Logarithmic Function

A *logarithmic function* is of the form

$$f(x) = \log_b x,$$

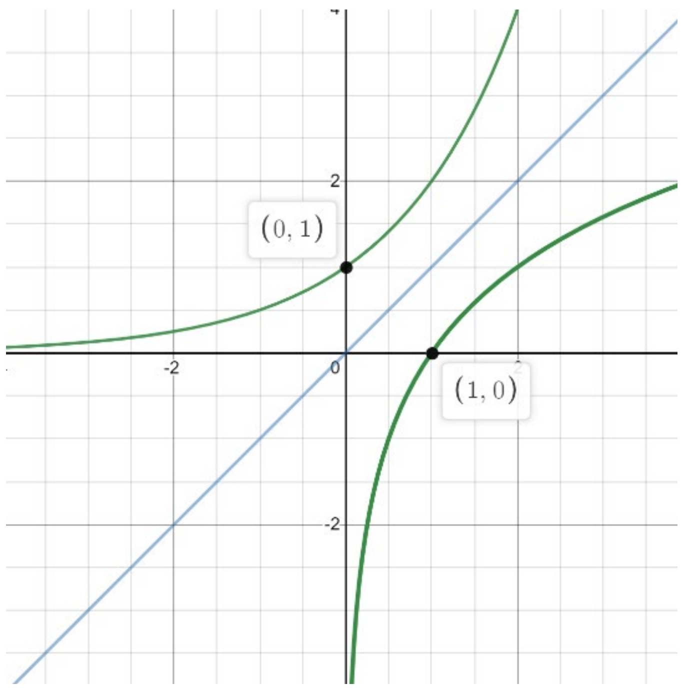
Where b is the base and $x > 0$, $b > 0$, and $b \neq 1$

Logarithmic and Exponential functions are *inverse functions*, which means that

$$y = \log_b x \text{ is the same as saying } b^y = x$$

and

$$y = b^x \ \log_b y = x \text{ is the same as saying } \log_b y = x$$



The properties of logarithms and exponents are related because they are inverse functions.

Properties of Exponents	**Properties of Logarithms**
$b^0 = 1$	$\log_b 1 = 0$
$b^1 = b$	$\log_b b = 1$
$b^{-1} = \frac{1}{b}$	$\log_b \frac{1}{b} = -1$
$b^x = b^x$	$\log_b b^x = x$ for all x, $b^{\log_b x} = x$ when $x > 0$
$b^m b^n = b^{m+n}$	$\log_b M \cdot N = \log_b M + \log_b N$
$\frac{b^m}{b^n} = b^{m-n}$	$\log_b \frac{M}{N} = \log_b M - \log_b N$
$(b^m)^p = b^{m \cdot p}$	$\log_b M^p = p \cdot \log_b M$

Special Logarithmic Functions

Common log: $\log_{10} x = \log x$

Natural log: $\log_e x = \ln x$

Change of base formula

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

Examples

$2^3 = 8 \leftrightarrow \log_2 8 = 3$

$e^x = 5 \leftrightarrow \ln 5 = x$

$4 \log x = 2 \leftrightarrow \log x^4 = 2 \leftrightarrow 10^2 = x^4$

Common Applications of Exponential Functions

Compound Interest
If $\$P$ is the initial deposit, and interest is paid n times per year at an annual rate of *r*, the amount A in the account after *t* years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Radioactive Decay
If A is the amount of radioactive material present at time *t*, A_0 was the amount present at $t = 0$, and h is the material's half-life, then

$$A = A_0 2^{-t/h}$$

Exponential Growth/Decay
If P is the population at some time *t*, P_0 is the initial population at $t = 0$, and *r* is the rate of growth/decay, then

$$P = P_0 e^{rt}$$

Common Applications of Logarithmic Functions

pH of a Solution
If [H+] is the hydrogen ion concentration in gram ions per liter, then

$$\text{pH} = -\log[H+]$$

Decibel Voltage Gain
If the output voltage to a device is E_0 volts and the input voltage is E_1, then the decibel dB gain is given by

$$\text{dB gain} = 20\log\frac{E_0}{E_1}$$

The Richter Scale
If *R* is the intensity of an earthquake on the Richter Scale, A is the amplitude (measured in micrometers) of the ground motion and *P* is the period (the time of one oscillation of the Earth's surface measured in seconds), then

$$R = \log\frac{A}{P}$$