Exponential & Logarithmic Functions

UVU Math Lab

An **exponential function** is of the form

$$f(x) = b^x,$$

where b is the base and

$b > 0$ and $b \neq 1$.

A **logarithmic function** is of the form:

$$f(x) = \log_b x,$$

where b is the base and

$x > 0, b > 0,$ and $b \neq 1$

Inverse Functions

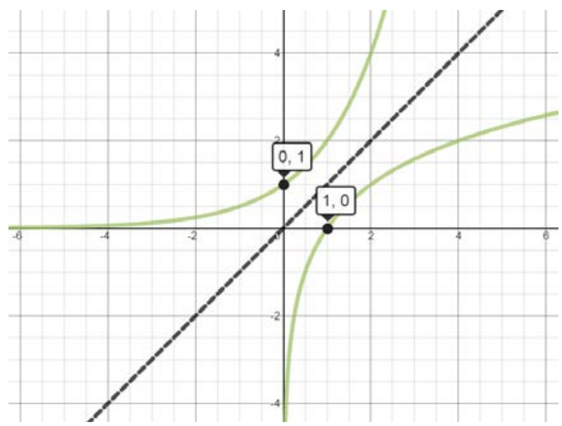
So we can say

$$b^y = x \Leftrightarrow \log_b x = y$$

Line of reflection: $y = x$

$\log_b x = y$

$b^x = y$



Properties of Exponents

$$b^0 = 1$$

$$b^1 = b$$

$$b^{-1} = \frac{1}{b}$$

$$b^x = b^x$$

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^p = b^{m \cdot p}$$

Notice the relationship.

Properties of Logarithms

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b \frac{1}{b} = -1$$

$$\log_b b^x = x \text{ and } b^{\log_b x} = x, \quad x > 0$$

$$\log_b M \cdot N = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^p = p \cdot \log_b M$$

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Examples:

$$2^3 = 8 \quad \leftrightarrow \quad \log_2 8 = 3$$

$$9^{\frac{1}{2}} = 3 \quad \leftrightarrow \quad \log_9 3 = \frac{1}{2}$$

$$(5)^{-2} = \frac{1}{25} \quad \leftrightarrow \quad \log_5\left(\frac{1}{25}\right) = -2$$

$$e^x = 5 \quad \leftrightarrow \quad \ln 5 = x$$

$$e^y = x^2 \quad \leftrightarrow \quad \ln x^2 = y \quad \leftrightarrow \quad 2 \cdot \ln x = y$$

Special Logarithmic Functions:

The Common Log: $\log_{10} x = \log x$

The Natural Log: $\log_e x = \ln x$

Change of Base Formulas:

$$\log_b x = \frac{\log x}{\log b} \text{ or } \log_b x = \frac{\ln x}{\ln b}$$

Common Applications of Exponential Functions:

Compound Interest:

If $\$P$ is the initial deposit, and interest is paid n times per year at an annual rate of r, the amount A in the account after t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Radio Active Decay:

If A is the amount of radioactive material present at time t, A_0 was the amount present at $t = 0$, and h is the material's half-life, then

$$A = A_0 2^{-t/h}$$

Exponential Growth:

If P is the population at some time t, P_0 is the initial population at $t = 0$, and r is the rate of growth, then

$$P = P_0 e^{rt}$$

Common Applications of Logarithmic Functions:

pH of a Solution:

If $[H^+]$ is the hydrogen ion concentration in gram-ions per liter, then

$$\text{pH} = -\log[H^+]$$

Decibel Voltage Gain:

If the output voltage to a device is E_0 volts and the input voltage is E_1, then the decibel dB gain is given by

$$\text{dB gain} = 20 \log \frac{E_0}{E_1}$$

The Richter Scale:

If R is the intensity of an earthquake on the Richter Scale, A is the amplitude (measured in micrometers) of the ground motion and P is the period (the time of one oscillation of the Earth's surface measured in seconds), the

$$R = \log \frac{A}{P}$$