

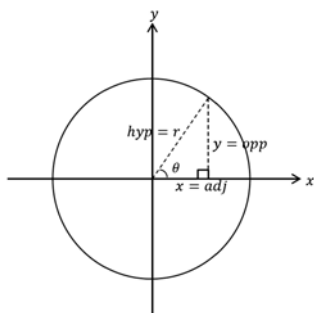
Trigonometric Basic Identities

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HINT: In many cases, we can use the **Reciprocal Identities** to rewrite expressions as functions of sine & cosine in order to more easily simplify, solve, or to reduce the amount of material to memorize (So, memorize the green information only.).

Definition of Trigonometric Functions:

Given a right triangle, where $0 < \theta < 90^\circ$,



Reciprocal Identities:

$$\begin{aligned} \sin \theta &= \frac{1}{\csc \theta} & \csc \theta &= \frac{1}{\sin \theta} \\ \cos \theta &= \frac{1}{\sec \theta} & \sec \theta &= \frac{1}{\cos \theta} \\ \tan \theta &= \frac{1}{\cot \theta} & \cot \theta &= \frac{1}{\tan \theta} \end{aligned}$$

$$\begin{aligned} \text{SOH} \quad \sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} & \csc \theta &= \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} \\ \text{CAH} \quad \cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} & \sec \theta &= \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} \\ \text{TOA} \quad \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} & \cot \theta &= \frac{\cos \theta}{\sin \theta} = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} \end{aligned}$$

where, $r = \sqrt{x^2 + y^2}$

Pythagorean Identities:

$$\begin{aligned} & \swarrow \sin^2 \theta + \cos^2 \theta = 1 \searrow \\ \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} &= \frac{1}{\sin^2 \theta} & \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta} \\ 1 + \cot^2 \theta &= \csc^2 \theta & \tan^2 \theta + 1 &= \sec^2 \theta \end{aligned}$$

Even & Odd Trig Functions:

$$\sin(-\theta) = -\sin \theta \text{ (odd)} \quad \cos(-\theta) = \cos \theta \text{ (even)}$$

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin \theta} = -\csc \theta \text{ (odd)}$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos \theta} = \sec \theta \text{ (even)}$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta} = -\tan \theta \text{ (odd)}$$

$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos \theta}{-\sin \theta} = -\cot \theta \text{ (odd)}$$

Co-function Formulas:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

See why this is true on Table of Trig Values.

Trigonometric Unit Circle

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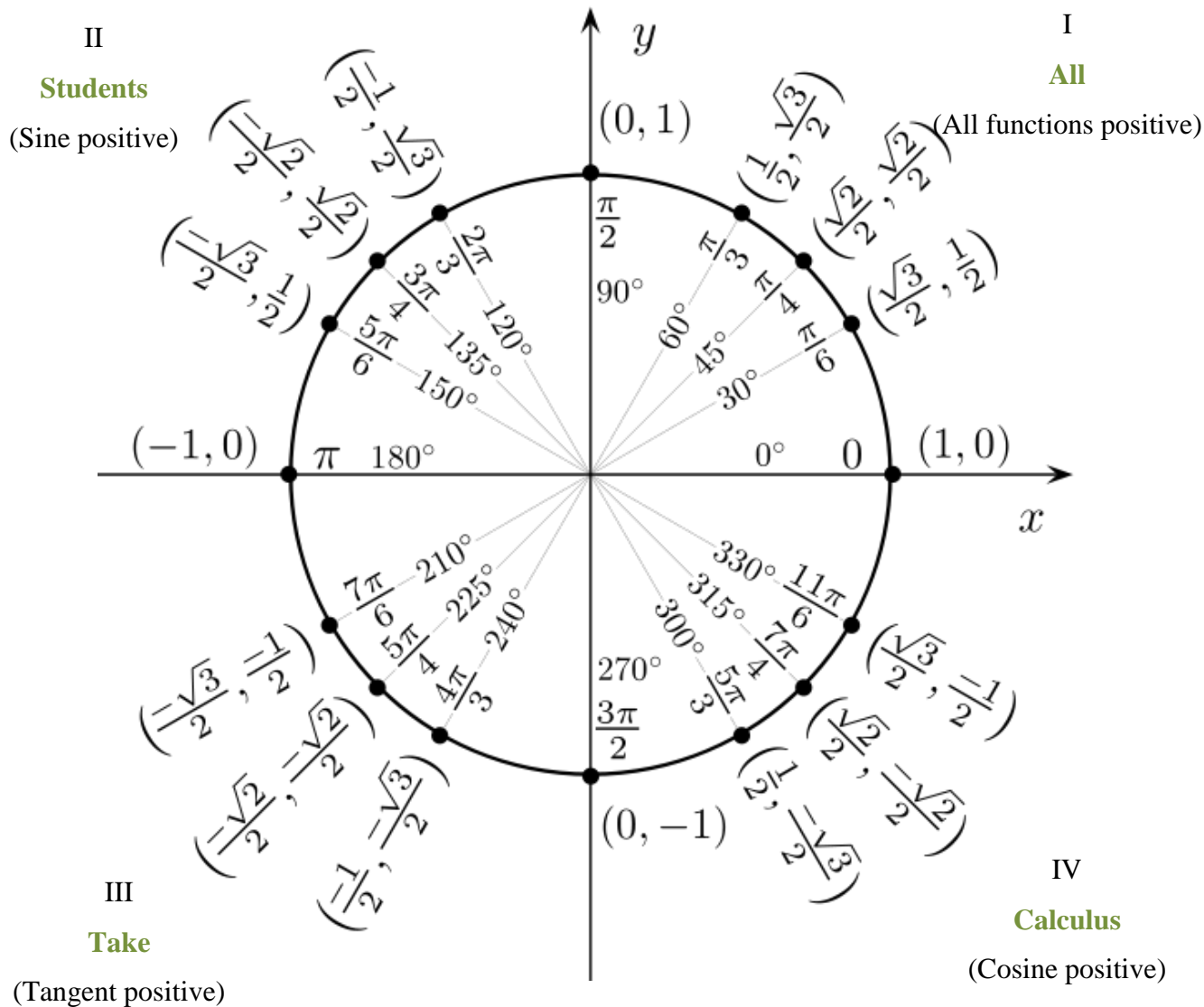


Table of Trigonometric Values of Common Angles of the Unit Circle:

Angle in Degrees / Radians	$\sin \theta$	$\cos \theta$	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
$30^\circ / \pi/6$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\frac{\sqrt{1}}{2}} = 2$	$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$	$\frac{\frac{\sqrt{1}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$	$\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{1}}{2}} = \sqrt{3}$
$45^\circ / \pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$	$\frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$
$60^\circ / \pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$	$\frac{1}{\frac{\sqrt{1}}{2}} = 2$	$\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{1}}{2}} = \sqrt{3}$	$\frac{\frac{\sqrt{1}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

More handouts like this are available at: www.uvu.edu/mathlab/mathresources/