Trigonometric Sum, Difference, Product Identities & Equations:

UVU Math Lab

Many of the following identities can be derived from the Sum of Angles Identities using a few simple tricks.

Sum of Angles Identities:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Let $\beta=\alpha$

$$\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$$

$$\cos(\alpha+\alpha)=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha$$

$$\tan(\alpha+\alpha)=\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$$

Simplify

Double-Angle Identities:

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

Use the Pythagorean Identity:

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$\cos 2\alpha=\cos^2\alpha-(1-\cos^2\alpha)$$

$$=2\cos^2\alpha-1$$

and

$$\cos 2\alpha=(1-\sin^2\alpha)-\sin^2\alpha$$

$$=1-2\sin^2\alpha$$

Let $\beta=-\beta$

Difference of Angles Identities:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Add Sum & Difference of Angles Identities:

Product to Sum Identities

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$+\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$$

Likewise, we can find:

$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$

$$2\cos\alpha\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$

Simply rearrange.

Power-Reducing Identities:

$$\cos 2\alpha=1-2\sin^2\alpha\rightarrow\sin^2\alpha=\frac{1-\cos 2\alpha}{2}$$

$$\cos 2\alpha=2\cos^2\alpha-1\rightarrow\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$$

$$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\frac{1-\cos 2\alpha}{2}}{\frac{1+\cos 2\alpha}{2}}=\frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

Take square root & divide α by 2.

Half-Angle Identities:

$$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Multiply by $\sqrt{\frac{1+\cos\alpha}{1+\cos\alpha}}$ or $\sqrt{\frac{1-\cos\alpha}{1-\cos\alpha}}$ to get

$$=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$$

$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

Product to Sum Identities:

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

The Sum to Product Identities take a little more manipulation.

Using the Product to Sum Identities, we let

$$x = \alpha + \beta \quad \& \quad y = \alpha - \beta$$

$$\alpha = \frac{x+y}{2} \qquad \beta = \frac{x-y}{2}$$

Plug in the new variables & simplify.

Sum to Product Identities:

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Formulas for Triangles:

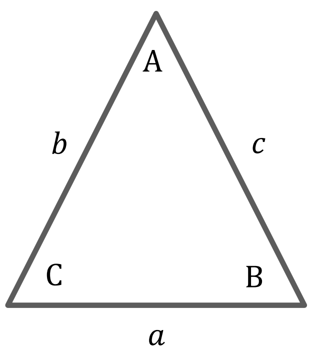
For any triangle, ABC…

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$



Area of a Triangle:

$$K = \frac{1}{2}ab\sin C \qquad K = \frac{a^2\sin B\sin C}{2\sin A}$$

Heron's Formula:

$$K = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+C}{2}$$