The valuation of legal claims using option pricing principles

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Abstract

This paper describes judgment on a legal claim as the product a binary option (as to liability) and a share-or-nothing option (as to damages). It considers how option pricing principles and the binary and share-or-nothing option characteristics affect the value of a legal claim prior to judgment (that is, the incentive to commence an action and/or to settle a claim prior to judgment). It also examines how option characteristics in the judgment affect the value of options that arise from the legal process itself.

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1. Introduction

A widely used method of valuing a legal claim to determine whether to bring an action on the claim or to accept a proposed settlement offer applies involves balancing an estimate of the probability of prevailing and the expected amount of the judgment in the event of a favorable verdict against the litigation costs the party must incur in order to pursue the claim. In some legal areas, statistical analysis of data from similar claims make detailed estimates possible. See, e.g., Jury Verdict Research Service

A developing literature considers how option pricing principles affect the value of legal claims. The focus of this research to date has been on options that arise due to the nature of a legal proceeding.

Cornell (1990) notes that incurring the expense to commence a lawsuit creates a number of “options”. The plaintiff may then initiate procedures inherent in the legal process, such as discovery, various types of motions, settlement and set the pace of the proceeding. Cornell notes that a plaintiff will not settle an action unless the settlement offer exceeds the sum of expected trial proceeds and the claim’s “option value”. That option value depends on plaintiff’s cost of continuing the claim and defense costs imposed on the other party. Cornell uses a decision tree to demonstrate how to decision whether to commence an action.

Grundfest and Huang (1996) develop a model based on a plaintiff’s staged investment in a lawsuit. Each investment of additional funds by a plaintiff imposes costs on a defendant and provides the parties with more complete information about the ultimate value of the judgment. The plaintiff can costlessly abandon the action at any
time. Considered in this manner, the litigation process is a compound real option; that is, an option which if exercised gives the owner another option, which if in turn exercised entitles the owner to another, and etc. until the final option is exercised generating its resulting payout, the judgment. Because of the real options that arise inherently from the legal process, option pricing principles must apply when valuing an as-yet-unfiled claim or one that is still in the midst of the legal process.

Using the option characteristics implicit in the legal process, Bebchuk (1996) shows that when litigation costs are incurred in stages, a threat to commence a lawsuit may be credible despite the fact that the lawsuit itself has negative expected value given the probability of prevailing, the expected amount of the judgment in the event of a favorable result and the total costs to bring and maintain an action. Blanton (1995) show how the procedural feature that allows a claimant to “opt out” after commencing an action affects incentives to initiate the action in the first place.

Nagareda (2002) characterizes a class member’s decision whether to accept a proposed class action settlement as a put option, which is exercised by taking the proposed settlement, rather than pursuing an individual claim. Klock (2002) describes Al Gore’s position after the 2000 presidential election as that of the holder of an out-of-the-money option, noting that in this instance the holder of the option also controlled the volatility of the underlying asset which directly affects the value of the option (an action to contest the result). Huang (1998) proposes that option characteristics implicit in a civil rights lawsuit suggest that option pricing principles, rather than statistical probability of prevailing, should be used to set attorneys’ fees.
This paper extends the use of option pricing principles in valuing legal claims by considering the option characteristics inherent in the judgment itself rather than those that arise as a result of particular characteristics of the procedure by which claims are adjudicated. Based upon facts established at trial and applicable legal principles, a trier of fact decides whether a plaintiff receives nothing or is entitled to a judgment. The judgment depends on the whether the facts shown establish a claim under the law and upon the damages established by the plaintiff. However, identical facts presented in two cases, even to the same trier of fact, will not necessarily result in identical judgments. Cooter and Rubinfield (1989) note that a judgment is selected by the trier of fact from a portfolio of judgments possible based upon the facts presented by the parties at trial. That is, the same facts in two different cases do not necessarily lead to the same judgment but will lead to a judgment selected from a distribution of possible judgments appropriate for those facts. Moreover, when the trier of fact is a jury, variation in jury composition can also create variability in the resulting judgment.

Since the distribution of possible judgments has a profile like that of the payoff on an option, the value of a legal claim prior to judgment (that is, the settlement value of a claim) must also option pricing. In addition, the value of an as-yet-unfiled claim is more appropriately determined using option pricing principles, rather than the probability of prevailing, the expected judgment in the event of success and the costs to maintain the action. Finally, an option created by the legal process is compound option (an option on an option) and must appropriately account for the option characteristics of the judgment itself.
The next section discusses applicability of option pricing principles generally and describes the two “exotic” options that relate to judgment on legal claims. Section 3 analyzes the the option characteristics inherent in the judgment on a legal claim. Sections 4 and 5 identify how changes in critical variables affect the option value of the legal claim, and how viewing the judgment itself using option pricing principles affects the analysis of options that arise due to procedural characteristics of the adjudication of claims. The final section concludes and suggests some possible extensions of the option approach to valuing legal claims.

2. Options and option pricing

The term “option” is used in two different senses in the legal literature. In one meaning, option refers to a choice between two alternatives, such as whether or not to initiate a legal action. When an underlying variable upon which the payoff depends has a finite number of different possible outcomes, it is not necessary to use option pricing for valuation. Each alternative can be individually valued and the overall decision can be evaluated using a decision tree.

In the other use of the term “option”, the underlying variable can take any of an infinite number of possible outcomes, with the option’s payoff value depending on that outcome. Such is the case with a financial option, such as an option on stock. The stock price upon which the payoff (and value prior to expiration) depends can take any of an
infinite number of possible variables.\textsuperscript{1} Valuing this option requires application of option pricing principles, such as those employed in the Black-Scholes option pricing model.

Regular call and put options are widely understood. However there are a variety of other instruments with option characteristics. These are sometimes referred to as “exotic” options. Two of these are useful when viewing legal claims as options, the binary (or digital) option and the share or nothing option.

A basic binary (or digital) option on a financial asset pays $1 when the market price of the asset exceeds the option’s strike price and $0 if it out of the money. Figure 1a presents a diagram of the payoff at expiration on a binary call. Prior to expiration, the value of a binary call option is:

\[ B = e^{-rt} N(d_2), \]

where \( e \) is the base of the natural logarithm, \( r \) is the risk free interest rate, \( t \) is the time to expiration of the option, \( S \) is the current market price of the underlying assets and \( \sigma \) is the standard deviation of returns on the underlying asset, \( X \) is the option’s strike price, and \( N(d_2) \) is the value of the standard normal cumulative distribution function at \( d_2 \). Partial derivatives of the binary option price with respect to \( S \), \( X \), \( r \), \( t \) and \( \sigma \) are presented in appendix 1.

A share or nothing call option on a financial asset pays the value of the underlying asset when in the money (that is, when the value of the underlying asset exceeds the strike price) and zero otherwise. Figure 1b diagrams the payoff on a share or nothing call option at expiration. Prior to expiration, the value of a share or nothing call is:

\[ C = S N(d_1), \]

where \( S \) is the current market price of the underlying asset, \( S \) is the current market price of the underlying asset,\textsuperscript{1}

\textsuperscript{1}The number of possible outcomes need not actually be infinite provided it is sufficiently large that valuing the option using decision tree analysis is not practical.
\[ d_1 = \frac{\ln\left(\frac{s}{Xe^{-r}}\right) + \sigma \sqrt{t}}{\sigma \sqrt{t}} \frac{1}{2}, \]

where \( X \) is the strike price, \( r \) is the risk free interest rate, \( t \) is the time to expiration of the option, \( \sigma \) is the standard deviation of returns on the underlying asset, \( N(d_1) \) is the value of the cumulative standard normal distribution evaluated at \( d_1 \).

Partial derivatives of the share or nothing call with respect to factors that determine its value are presented in Appendix 2.

### 3. Option characteristics of judgment in a civil action

The judgment on a civil claim depends on the findings by the trier of fact in two distinct areas, liability and damages.\(^1\) To establish liability, plaintiff must adequately prove the elements of the claim. The trier of fact decide whether plaintiff met the burden of proof as to the defendant’s liability. This decision can be modeled as a binary option, either liability is established (payoff on the option is $1) or not (payoff on the option is $0). The damages portion of the case is a share or nothing call option. When the plaintiff has established damages sufficiently to justify an award of damages, the option is “in the money” and the payoff is the amount of damages proved. When the plaintiff fails to prove damages, the option expires out of the money and no damages are awarded. The value of the legal claim at the time of judgment is the product of the payoff on the binary, liability, option and the payoff on the share or nothing, damages, option. Since the payoff at judgment equals the product of the payoff on these two options, the value of the claim prior to judgment equals the product of the two option prices prior to expiration.

\(^1\)Criminal cases also involve two components, guilt and sentencing. This paper focuses on civil litigation but the option pricing principles described apply also to criminal actions.
As to liability, it is more accurate to view the liability portion of a civil action as an option than to base the action’s expected value on the average probability of prevailing at trial. While for many similar cases, statistics may indicate that the plaintiff prevails a certain percentage of the time, in each individual case, the plaintiff either establishes liability, making the probability of recovery in that case is 100%. or fails to do so, in which case the probability of recovery is zero.\textsuperscript{2} Using option pricing for this portion of the judgment better reflects the actual outcome than does an average probability of recovery.

In a financial option, the market price of the underlying asset (e.g., a stock price, an interest rate, or an exchange rate) which determines the option’s value at expiration is readily observable but changes up until the option expires. Judgment on liability depends instead on many facts that must be established and on arguments presented to the trier of fact. The adjudicator then makes the determination of liability in light of its understanding and interpretation of the facts relative to the law applicable to the claim. Viewing liability as a binary option differs from a basic financial binary option in two ways. First, there are multiple underlying “assets” in the liability option rather than a single underlying price that determines payoff on the financial binary option. The decision on liability depends on each necessary fact being established; this “option” requires that each “underlying asset” exceeds its strike price in order for the option to be “in the money.” A financial option which depends for its value, and ultimately its payoff, on multiple underlying assets is referred to as a multicolor rainbow option.

\textsuperscript{2}Even where comparative negligence replaces an all or nothing rule, the decision on liability is better modeled with a spread (a combination of the owner’s position in a call with a lower strike price and the writer’s position in a call with a higher strike price) than by a measure of probability of recovery.
valuation of these options is an extension of the valuation of options that depend on only a single price. For purposes of this paper, it is sufficient to think of the underlying “asset” for the liability “option” as a vector of variables that taken together determine liability.

The second difference between a financial binary option and option pricing for the liability determination relates to price movement of assets that underlie financial options and the fact that there is no “price” for the liability “option” and the facts upon which liability is based do not change from the time the action commences until judgment. However, the parties’ knowledge of the relevant facts may be incomplete at commencement of a legal action, and the same facts may be subject to differing interpretations. Consequently, at the time of judgment, the trier of fact in essence draws a “value” from a distribution of all possible outcomes that have as their starting point the plaintiff’s understanding of the facts at the time the action is filed. This is not entirely unlike the price movement on a financial asset that leads to a value at expiration drawn from a distribution of possible values that is known at the creation of the option.

The option interpretation of the damages portion of the judgment is similar. A plaintiff proves damages by adducing facts and persuading the trier of fact that the facts established support a certain level of damages. In different cases, the same underlying facts may support different judgments. Even when liability is adequately established and proof presented regarding damages sustained, the actual amount of the judgment is drawn from a distribution of possible judgments supported by those facts. The amount of the judgment may be zero, if the plaintiff fails to adequately prove damages, or many times the out-of-pocket costs the plaintiff incurs due to the defendants actions. The fact that the
amount of the verdict depends on some underlying facts and that a variety of possible outcomes including no recovery are possible indicates that the damages portion of a legal claim should also be evaluated as a derivative. Underlying facts that affect the amount of damages include such things as out-of-pocket costs, economic loss or long term effects, and the extent to which the damaged party has mitigated its losses. The decision regarding damages may be viewed as a share-or-nothing option.

As is the case with the liability portion of the judgment, the amount of damages awarded depends on multiple facts and not the market price of a single underlying variables. The underlying asset for the damages option is a vector of facts each of which must be in the money in order for the option to have non-zero value at expiration.

The value of the judgment is the product of the payoff on the liability option (zero or one) and the payoff on the damages option. Both options must be “in the money” at expiration (that is, at judgment) for the verdict to be given in favor of the plaintiff. Prior to judgment, value of the legal claim, the value plaintiff uses to determine whether to invest the sums required to commence the action and the value for which the parties may settle the action prior to judgment, is the product of the values of its two component options, $J = BC$.

4. Effects of variables on value of litigation

Partial derivatives for the option value of a legal claim indicate how the factors that determine value of the claim affect the pre-filing value and/or settlement value. Those partial derivatives are presented and briefly discussed in this section. (To
distinguish between factors that affect liability and those that affect damages, a “ˆ” is used to identify those variables associated with the liability determination.)

The partial derivatives with respect $S$ and $S$ indicate how the strength of a case on liability or damages affects the value of the claim. As expected, the stronger the facts in support of either liability or damages, the higher the value of the claim.

$$\frac{\partial J}{\partial S} = C \frac{e^{-rt} N'(d_2)}{S\sigma \sqrt{t}} > 0$$

and

$$\frac{\partial J}{\partial S} = B \left( N(d_1) + \frac{N'(d_1)}{\sigma \sqrt{t}} \right) > 0.$$

Partial derivatives with respect to $X$ and $X$ indicate how the plaintiff’s burden of proof affects the value of the claim. These indicate that the higher the burden of proof, the lower the value of the claim.

$$\frac{\partial J}{\partial X} = -C \frac{e^{-rt} N'(d_2)}{X\sigma \sqrt{t}} < 0$$

and

$$\frac{\partial J}{\partial X} = -B \frac{SN'(d_1)}{X\sigma \sqrt{t}} < 0.$$

Partial derivatives with respect to $\sigma$ and $\sigma$ indicate how greater variability of outcomes based on the facts affect the value of the claim. These are not monotonic.

$$\frac{\partial J}{\partial \sigma} = -C \frac{e^{-rt} N'(d_2)\hat{d}_1}{\hat{\sigma}} \text{ may be greater or less than zero (} \hat{d}_1 = \hat{d}_2 + \hat{\sigma} \sqrt{t} \text{).}$$

This is usually negative; settlement value is inversely related to the variability of facts that tend to establish liability. The greater the variability, the lower the claim value.
However, the sign is reversed when \( d_1 \) is negative. This occurs when \( S \) is small relative to \( X e^{-\sigma t} \), provided \( \sigma \) and \( t \) are sufficiently large. In other words, uncertainty about the facts that must be proved to establish liability increases the pre-filing and/or settlement value when given the current facts the option is out of the money and the variability and/or the time to judgment is large. Figure 2a presents the relationship between this partial derivative and \( S \).

\[
\frac{\partial J}{\partial \sigma} = -B \frac{SN'(d_1)d_2}{\sigma} \text{ may be greater or less than zero (} d_2 = d_1 - \sigma \sqrt{t} \text{).}
\]

Settlement value is also usually inversely related to the variance of facts that establish the amount of damages, greater variability reduces the value of the claim. The sign reverses when \( d_2 \) is negative, that is, when \( S \) is small relative to \( X e^{-\sigma t} \) provided \( \sigma \) and \( t \) are sufficiently large. Uncertainty about the facts that establish damage reduce settlement value unless the option is out of the money (the case for damages is weak) and variability of those facts and/or time to judgment is large. Figure 2b presents the relationship between this partial derivative and \( S \).

The relationship between claim value and the risk free interest rate can be positive or negative.

\[
\frac{\partial J}{\partial r} = t \left( \frac{BSN'(d_1)d_2}{\sigma \sqrt{t}} + \frac{S e^{-\sigma t} N'(d_2)}{\hat{\sigma} \sqrt{t}} - BC \right) \text{ may be greater or less than zero.}
\]

It is generally positive except the when \( \sigma \) and \( t \) are large, it may be negative for some values of \( S, S, X \) and \( X \).

The partial derivative with respect to \( t \) indicates how the value of the claim is affected by the amount of time that will elapse before judgment.
\[
\frac{\partial J}{\partial t} = -rBC - BSN'(d_1)\left(\frac{d_2}{2t} - \frac{r}{\sigma \sqrt{t}}\right) - Ce^{-rt}N'(d_2)\left(\frac{d_1}{2t} - \frac{r}{\sigma \sqrt{t}}\right)
\]
may be greater or less than zero.

The relationship between time and claim value is positive whenever the option is out of the money. When the case for liability or damages is weak, the longer the time to judgment, the greater the value of the claim. When the option is in the money (the case for liability and damages is strong, a longer time until the case is decided may reduce the value of the claim when \(X\), \(X\), \(\sigma\), or \(r\) are low. That is, when there is a low required threshold of proof to establish liability of damages, when there is little uncertainty about the facts supporting liability or damages, a longer time to judgment negatively affects the value of a claim.

5. Implications for procedural options

An option that arises as a result of procedural aspects of litigation case is a compound options (that is, an option which has an option as its underlying asset).

Valuing a procedural option such as the right to abandon after discovery having only spent a portion of the total cost to prosecute a claim to judgment must account for the fact that the judgment itself has option characteristics. One example of this is how the variance of the facts affects the value of the procedural option. If the final judgment did not have option characteristics, the claim’s value would always be positively correlated with the variance of the judgment. Thus even if the expected value of the claim is low because a plaintiff is unlikely to prevail, the option value may be positive because of the positive correlation between option price and variance. When the underlying asset (the
expected judgment J) does not have option characteristics, the value of the procedural option, P, is positively correlated with the J’s variance, \( \frac{\partial P}{\partial \sigma_J} > 0 \). However, if the option characteristics of the judgment are also considered, the value of the procedural option may be positively or negatively correlated with the variance of the judgment, \( \frac{\partial P}{\partial \sigma} = (\frac{\partial P}{\partial J})(\frac{\partial J}{\partial \sigma}) \). The partial derivative of the procedural option with respect to the judgment, J, is positive. But the expression is signed by the second term, the partial of the judgement with respect to the variance of facts establishing liability or damages. That may be positive or negative.

6. Conclusion

Judgments on legal claims have properties that make them susceptible to valuation using option pricing principles. The liability determination is a binary option the pre-judgment value of which depends on the facts tending to establish liability, the plaintiff’s burden of proof, the level of uncertainty that the facts support a determination of liability, the risk free interest rate and the time until judgment. The legal determination of the amount of damages is a share or nothing option. Its value depends on the facts establishing the amount of damages and the uncertainty inherent in those facts, the burden of proof with respect to damages, the risk free interest rate and the time to judgment. The paper develops the relationships between these variables and the pre-judgment value of the claim, and shows that valuing claims using statistical likelihood of prevailing and average awards is inadequate. Rather, claims must be valued as the product of the binary, liability option and the share or nothing, damages option should form the basis for evaluating a claim prior to filing or for determining the sufficiency of a
proposed settlement. Further, recognition that options inherent in procedural aspects of litigation are more complex because they are in fact compound options, and the relationship between the liability and damages options and the variables that determine their values is more complex than that which exists between a simple call (or put) and its determinants.

Approaching the value of legal claims in this manner may provide additional insights regarding policy decision to alter the burden of proof, to limit damages, or otherwise manage the outcome of litigations. It also provides another means to evaluate how changes in litigation procedures are likely to affect incentives to sue or settlement after initiation of litigation.
Appendix 1: Partial derivatives of a binary option, B.

\[
\frac{\partial B}{\partial S} = \frac{e^{-rt}N'(d_2)}{S\sigma\sqrt{t}} > 0
\]

\[
\frac{\partial B}{\partial X} = -\frac{e^{-rt}N'(d_2)}{X\sigma\sqrt{t}} < 0
\]

\[
\frac{\partial B}{\partial r} = -t \left( B - \frac{e^{-rt}N'(d_2)}{\sigma\sqrt{t}} \right)
\]
may be greater or less than zero. It is negative when t or \( \sigma \) is large and when the option is deep in the money.

\[
\frac{\partial B}{\partial t} = -rB - e^{-rt}N'(d_2) \left( \frac{d_1}{2t} - \frac{r}{\sigma\sqrt{t}} \right)
\]
may be greater or less than zero. It is negative unless \( \sigma \) or \( t \) is very small.

\[
\frac{\partial B}{\partial \sigma} = -\frac{e^{-rt}N'(d_2)d_1}{\sigma}
\]
may be greater or less than zero. It is negative when \( d_1 \) is positive, that is, when the current value exceeds the present value of the strike price provided \( t \) and \( \sigma \) are not too large, positive when the options is sufficiently out of the money.
Appendix 2: Partial derivative of share or nothing option, C.

\[ \frac{\partial C}{\partial S} = \frac{N'(d_1)}{\sigma \sqrt{t}} + N(d_1) > 0 \]

\[ \frac{\partial C}{\partial X} = -\frac{SN'(d_1)}{X \sigma \sqrt{t}} < 0 \]

\[ \frac{\partial C}{\partial r} = \left( \frac{SN'(d_1)}{\sigma \sqrt{t}} \right) > 0. \]

\[ \frac{\partial C}{\partial t} = -SN'(d_1) \left( \frac{d_2}{2t} - \frac{r}{\sigma \sqrt{t}} \right) \]

may be greater or less than zero. It is negative unless \( \sigma \) or \( t \) is very small.

\[ \frac{\partial C}{\partial \sigma} = -\frac{SN'(d_1)d_2}{\sigma} \]

may be greater or less than zero. It is negative when the option is in the money or slightly out of the money, positive if the option is sufficiently in the money.
References:


Figure 1.

a. Payoff on a binary option

![Graph showing the payoff of a binary option.](image)

Value of underlying asset, $S$

b. Payoff on a share or nothing option

![Graph showing the payoff of a share or nothing option.](image)

Value of underlying asset, $S$
Figure 2

a. $\frac{\partial J}{\partial \hat{\sigma}}$ as a function of $\hat{S}$.

b. $\frac{\partial J}{\partial \sigma}$ as a function of $S$. 