Determinants of the Real Exchange Rate in a Small Open Economy: Evidence from Canada

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Abstract:
This study develops a theoretical monetary model of the real exchange rate and shows that over the long run the real exchange rate is a function of real money supply, domestic and foreign interest rate, real GDP, real government expenditure, deficit per GDP, domestic and foreign outstanding debt per GDP, domestic and foreign externally-financed debt per GDP and commodity price. The model was tested on Canadian data (1972Q1-2010Q3 period). It was found that all variables, except real money supply, domestic and foreign interest rate and domestic externally-financed debt have a statistically significant impact on the real exchange rate in Canada. However, the domestic fiscal variables do not have any impact on the real exchange rate over the short run. The change in interest rate, the growth of money supply, the commodity price and the US debt per GDP have a negative impact on the growth of the real exchange rate over the short run. The impulse responses of the real exchange rate to all shocks, except money supply and real income, are permanent and the stronger contributor to forecast error variance is the commodity price index.

Keywords: Real exchange rate, monetary policy, debt, deficit and foreign financing

JEL Codes: F31, F40 and F41
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1. Introduction

The real exchange rate, as a measure of price-cost competitiveness, could lose its significance for explaining trade flows if its fundamental determinants are not known. Macro-fundamental variables are determinants of the exchange rate according to the standard exchange rate models. If central banks follow Taylor rules it is argued (e.g., Mark (2009)) that fundamental determinants of the exchange rate can include the relative expected inflation gaps and relative output gaps. According to Mark (2009), the fundamental variables and rational expectations poorly explain the real exchange rate in the current literature.

According to Devereux’s (1997) survey result, under short-run sticky prices and high capital mobility (Mundell-Fleming-Dornbusch type models), fluctuations of the nominal exchange rate will be translated one for one into the real exchange rate. In such models, the nominal exchange rate is more volatile than its underlying fundamentals over the short run, but as prices do adjust over the long run the real exchange rate converges to its long-run equilibrium level in the same proportion. Furthermore, over the long run, the real exchange rate converges to Purchasing Power Parity (PPP) implying that fundamental macro variables do not have any impact on the real exchange rate.

An alternative view is that prices are fully flexible and the real exchange rates adjust to real disturbances and fiscal policies. The monetary policy cannot influence the real exchange rate, as in this equilibrium view both price and nominal exchange rate adjust in the same proportion to any monetary shock, Devereux (1997). However, it is possible that prices of all goods be set in local currency and adjust only over time. In this situation we can have two kinds
of price rigidity: first, staggered price setting, where prices are set in advance for some given future periods; second, partial adjustment price setting mechanism, which is due to Calvo (1983), see, e.g., Kollman (1997) who provides evidence for the real exchange rate variability from G7 countries.

Under no barrier to trade and rational expectations, we would expect PPP to hold over the long run irrespective of short run flexible or sticky prices. However, there is a lack of empirical evidence for supporting PPP even over the long run, see, e.g., Devereux (1997) and the quoted literature within. One possible explanation for such a deviation from PPP is that the nominal exchange rate and prices react to macro fundamentals in different proportions so that fluctuations in macro variables, specially monetary and fiscal variables, lead to fluctuations in the real exchange rate, especially over the long run. The objective of this paper is to develop and test a model of the real exchange rate which can be used to investigate this explanation.

According to some studies, the fluctuations in the real exchange rate stem mostly from external factors or differences between external and internal factors, e.g., Reinhart et al. (1993) and Del Negro and Obiols-Hums (2001), Kandil et al. (2007) and Hamori and Hamori (2011). Some studies use one or more fiscal variables as well as some macro variables in the determination of the real exchange rate. Among those studies: Edwards (1988), Canzoneri et al. (2003), Égert et al. (2006), Candelon et al. (2007), Kim and Roubini (2008), Müller (2008), Galstyan and Lane (2009), Cayen et al. (2010) and Etta-Nkwelle, et al. (2010). Furthermore, some studies ignored completely most or all of the fundamental macro variables, especially fiscal variables. For example, see Schlagenhauf and Wrase (1995), Devereux (1997), Kim (2007), Morales-Zumaquero (2006) and Uz and Ketenci (2010).
In this paper, I constructed a real exchange model which can verify the impact of both fiscal and monetary variables, including all relevant macro-fundamental variables. To the best of my knowledge, no such model exists in the current literature. This is the first contribution of this paper.

It should also be mentioned that in estimating long-run cointegration relationships none of the existing literature allowed the short-run dynamics of the system to adjust for the factors (i.e., financial/economic crises, policy regime changes and other exogenous changes) which can influence the short-run dynamics of the system. Kia (2006b) shows that the result would be biased if we ignore this fact.

The model is estimated on Canadian data, while I allowed the short-run dynamics of the system to incorporate the implementation of inflation targeting by the Bank of Canada and the Department of Finance, the Free Trade and Nafta agreements as well as the recent US financial crisis. It was found that, contrary to the existing literature, e.g., Mark (2009), and the literature within, the fundamental variables perform very well over the long run. This is another contribution of this study. Moreover, it should be noted that Canada, as Table 1 reports, has the highest degree of openness among the G7 countries. Therefore, the Canadian data are very good observations for testing the model.

The next section describes the model and is followed with a section on data and long-run estimation results. Section 4 reports short-run dynamics models. Section 5 analyzes the impulse response functions and is followed by concluding remarks.
2. The Model

Assume a representative in the economy has preferences given by

\[
E \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, c_t^*, g_t, k_t, m_t, k^* m^*_t) \right\},
\]

where

\[
U(c_t, c_t^*, g_t, k_t, m_t, k^* m^*_t) = (1-\alpha)^{-1} (c_t^{\alpha_1} c_t^*^{\alpha_2} g_t^{\alpha_3})^{1-\alpha}
+ \xi (1-\eta)^{-1} [(m_t/k_t)^{\eta_1} (m^*_t/k^*_t)^{\eta_2}]^{1-\eta}.
\]

Furthermore, \(\alpha_1, \alpha_2, \alpha_3, \alpha, \eta_1, \eta_2, \eta\) and \(\xi\) are positive parameters and \(0.5<\alpha<1\) and \(0.5<\eta<1\).

The latter assumption \((0.5<\alpha<1\) and \(0.5<\eta<1\)) is needed to ensure a standard demand for money. Since none of the following results is sensitive to the magnitude of \(\alpha_1, \alpha_2, \alpha_3, \eta_1\) and \(\eta_2\), for the sake of simplicity, we assume these parameters are all equal to one. Here \(c_t\) and \(c_t^*\) are single, non-storable, real domestic and foreign consumption goods, respectively. \(m_t\) and \(m_t^*\) are the holdings of domestic real \((M/p)\) and foreign real \((M^*/p^*)\) cash balances, respectively. \(E\) is the expectation operator, and the discount factor satisfies \(0<\beta<1\). The real government expenditure on goods and services \((g)\) is assumed to be a “good”. In the specification of the utility function \((1)\), for the sake of simplicity, following Cox (1983), Drazen and Helpman (1990), Hueng (1999) and Kia (2006a) among many others, we assume that the total output is exogenously given. In other words, we assume labor is supplied inelastically. Note that none of the results will be affected if we relax this assumption.\(^1\)

\(^1\) If we relax the assumption that labor is supplied inelastically (income is exogenous) then we need to add an extra term, say, \(-(1-\eta_3)^{-1}(N_t)^{\eta_3}\) in the utility function, where \(N\) is hours worked and \(\eta_3 \geq 0\) represents Frisch labor supply elasticity. In such a case, one can easily verify that none of our results will be different.
Including government expenditure in preferences is based on the assumption that individuals benefit from government services, say, clean and safe roads, foods which have been inspected, etc. provide a higher utility to consumers. Alternatively, following the literature, we can consider $g$ as public demand for public goods. In fact, allowing consumer preferences to depend on government spending is not new in the literature, see, Kia (2006a) and references within. Following Sidrauski (1967), it is assumed services of money enter the utility function. Furthermore, following Stockman (1980), Lucas (1982), Guidotti (1993), Hueng (1999) and Kia (2006a), it is assumed that purchases of domestic and foreign goods are made with domestic and foreign currencies, respectively, and, therefore, the services of both domestic and foreign currencies enter the utility function. Let us choose the units in such a way that the services of domestic money $S$ is equal to $m$ and the services of foreign money $S^*$ is equal to $m^*$. Note that one can simply show that none of the results given in this paper will change if instead of Sidrauski’s services of money in the utility function we assume a shopping time or cash-in-advance model.

Following Kim (2000) and Kia (2006a), variable $k_t$, which summarizes risk associated to holding domestic money, is also included. However, in contrast with Kim, we assume variable $k$ is a function of anticipated fiscal variables over the long run and policy and political regime changes over the short run. Furthermore, in contrast to Kim and Kia we assume $k_t$ also includes the negative risk. Specifically, we postulate that over the long run:

$$\log(k_t) = k_0 \text{ defgdp}_t + k_1 \text{ debtgdp}_t + k_2 \text{ fdgdp}_t + k_3 \log(\text{com}_t).$$ (2)

It is assumed that the short-run dynamics of the risk variable $[\log(k)]$ includes a set of interventional dummies which account for economic crisis, innovations as well as policy regime changes which influence services of money. Variables defgdp, debtgdp, fdgdp and com are real
government deficit per GDP, the government debt outstanding per GDP, the government foreign-financed debt per GDP and the commodity price, respectively. We assume government debt pays the same interest rate as deposits at the bank (i.e., R). In a risky environment agents substitute real or interest-bearing assets for money. For example, as the government deficit per GDP increases agents perceive higher future taxes or money supply (inflation). At the same time, the higher is the outstanding government debt relative to the size of the economy, the riskier the environment will be. Individuals may hold these bonds to bridge the gap between the future labor income and expenditures, including tax expenditures. Consequently, we hypothesize constant coefficients $k_0 > 0$ and $k_1 > 0$. Furthermore, an increase in the amount of government debt held by foreign investors may be considered a cause for future devaluation of the domestic currency. Specifically, when a large amount of government-issued bonds is held by foreign investors there is always the risk that these investors decide to dump those bonds. This action would result in an increase in the supply of the country’s currency and so a devaluation of the currency. Consequently, the demand for domestic money may fall if a large portion of the government debt is financed externally, implying $k_2 > 0$. Finally, since Canada is a commodity oriented country, as the commodity price goes up the risk associated with holding Canadian dollars, everything else being constant, will fall, implying that $k_3 < 0$.

We also assume $k^*$ summarizes the risk associated with holding foreign currency, say $\$US$.

$$\log(k^*_t) = k_0^* \text{debtgdp}_t^* + k_1^* \text{fdgdp}_t^*.$$  \hspace{1cm} (3)

Variables $\text{debtgdp}_t^*$and $\text{fdgdp}_t^*$ are the outstanding foreign country debt per foreign GDP and the internationally-foreign government financed debt per foreign GDP, respectively. A higher foreign debt is assumed to be associated to future monetization of debt and a lower value of
foreign currency (i.e., a lower demand for foreign currency) and, consequently, $k_{0} > 0$. Similar to the domestic currency, an increase in the amount of government debt held by international investors/governments may be considered a cause for the future devaluation of the foreign currency, which implies $k_{1}^{*} > 0$.

Similar to Equation (2), we assume Equation (3) is held subject to a short-run dynamics of the system. Furthermore, it is assumed that the short-run dynamics of the risk variable associated to holding foreign currency $[\log(k^{*})]$ includes a set of interventional dummies which account for economic crisis, political changes and policy regime changes which influence the value of foreign money.

Assume also that the US dollar represents foreign currency. Given g, defgdp, debtgdp and fdgdp, the consumer maximizes Equation (1) subject to the following budget constraint:

$$\frac{\tau_{t}}{c_{t}} + \frac{\pi_{t}}{c_{t}} + \frac{m_{t-1}}{c_{t}} + \frac{q_{t}}{c_{t}} (\frac{1}{1 + \pi_{t}^{*}}) \frac{m_{t-1}^{*}}{c_{t}} + \frac{d_{t-1}}{c_{t}} + \frac{q_{t}}{c_{t}} (\frac{1}{1 + \pi_{t}^{*}}) \frac{d_{t-1}}{c_{t}} + \frac{c_{t}}{c_{t}} + \frac{q_{t}}{c_{t}} \frac{c_{t}}{c_{t}} + \frac{m_{t}}{c_{t}} + \frac{q_{t}}{c_{t}} \frac{m_{t}^{*}}{c_{t}} + \frac{d_{t}}{c_{t}} + \frac{q_{t}}{c_{t}} \frac{d_{t}}{c_{t}}^{*},$$  \hspace{1cm} (4)

where $\tau_{t}$ is the real value of any lump-sum transfers/taxes received/paid by consumers, $q_{t}$ is the real exchange rate, defined as $E_{t} p_{t}^{*}/p_{t}$, $E_{t}$ is the nominal market exchange rate (domestic price of foreign currency), $p_{t}^{*}$ and $p_{t}$ are the foreign and domestic price levels of foreign and domestic goods, respectively, $y_{t}$ is the current real endowment (income) received by the individual, $m_{t-1}^{*}$ is the foreign real money holdings at the start of the period, $d_{t}$ is the one-period real domestically financed government debt which pays $R$ rate of return and $d_{t}^{*}$ is the real foreign issued one-period bond which pays a risk-free interest rate $R_{t}^{*}$. Assume further that $d_{t}$ and $d_{t}^{*}$ are the only two storable financial assets.

Maximizing the preferences with respect to $m$, $c$, $m^{*}$, $c^{*}$, $d$ and $d^{*}$, and subject to budget constraint (4) for the given output and fiscal variables, will yield:
\[ c_t^* = c_t q_t^{-1}, \text{ or} \]
\[ \log(c_t^*) = \log(c_t) - \log(q_t). \]
\[ m_t^* = k_t^* \left( \frac{\eta - 1}{\eta} \right) \left( \frac{R_t^*/1 + R_t^*}{1 + R_t^*} \right)^{-1} \left( c_t^{1 - 2\alpha} g_t^{1 - \alpha} q_t^{\alpha} \right)^{1/\eta} \xi^{1/\eta} \xi_1 (k_t^{-1} m_t) \left( 1 - \eta \right)^{1/\eta}, \]
\[ \log(m_t^*) = \left( \frac{\eta - 1}{\eta} \right) \log(k_t^*) - \eta^{-1} \log(R_t^*/1 + R_t^*) - \eta^{-1} (1 - 2\alpha) \log(c_t) - \eta^{-1} \alpha \log(q_t) - \eta^{-1} (1 - \alpha) \log(g_t) + \eta^{-1} \log(\xi) + (\eta^{-1} - 1) \log(m_t) - (\eta^{-1} - 1) \log(k_t). \]

As we can see from Equation (6), for \( \eta < 1 \) foreign risk associated with holding foreign currencies results in a reduction in demand for foreign currency. Using equations (5) and (6), and assuming the domestic real consumption \( (c_t) \) is some constant proportion \( (\omega) \) of the domestic real income \( (y_t) \), where for simplicity we assume \( \omega = 1 \), we will have:
\[ \log(m_t) = m_0 + m_1 i_t + m_2 \log(y_t) + m_3 \log(g_t) + m_4 \log(k_t) + m_5 \log(q_t) + m_6 i_t^* + m_7 \log(k_t^*), \]
where, \( i_t^* = \log(R_t^*/1 + R_t^*) \) and \( i_t = \log(R_t/1 + R_t) \), using \( 0.5 < \alpha < 1 \) and \( 0.5 < \eta < 1 \) we will have:
\[ m_0 = -1 / (1 - 2\eta) \log(\xi) > 0, \ m_1 = \eta(1 - 2\eta)^{-1} < 0, \ m_2 = (1 - 2\eta)^{-1} (1 - 2\alpha) > 0, \]
\[ m_3 = - (1 - \alpha) < 0, \ m_4 = (1 - 2\eta)^{-1} (1 - \eta) < 0, \ m_5 = - (1 - 2\eta)^{-1} [(\eta - 1)\alpha - (1 - \alpha)] < 0 \]
\[ m_6 = - (\eta - 1) (1 - 2\eta)^{-1} < 0, \text{ and } m_7 = (1 - 2\eta)^{-1} (1 - \eta) < 0. \]

Note that the coefficient of both \( k \) and \( k^* \) is negative, implying that both domestic and foreign risks associated with holding domestic and foreign currencies reduce demand for domestic currency. This is because, as we can see from Equation (6), the demand for domestic currency \( (m) \) has a positive relationship with demand for the foreign currency \( (m^*) \). Therefore, as \( k^* \) goes up, \( m^* \) will fall which results in a fall of \( m \).

\footnote{The full derivation of Equation (6) is available upon request.}
At the equilibrium we will have \( \log(m_t) = \log(m^s_t) \), where \( m^s \) is the supply of money.

Substituting \( \log(m^s) \) for \( \log(m) \) in Equation (7) and also equations (2) and (3) for \( \log(k) \) and \( \log(k^*) \) in Equation (7) and solving for \( \log(q_t) \) result in:

\[
l_q = \beta_0 + \beta_1 \log(m^s_t) + \beta_2 i_t + \beta_3 y_t + \beta_4 \log(gdp_t) + \beta_5 \text{defgdp}_t + \beta_6 \text{debtgdp}_t + \beta_7 \text{fdgdp}_t + \beta_8 \text{lcom}_t + \beta_9 i^*_t + \beta_{10} \text{debtgdp}^*_t + \beta_{11} \text{fdgdp}^*_t + \nu_t, \quad (8)
\]

where \( lX = \log(X) \), \( \beta_0 = -m_0/m_5 > 0 \), \( \beta_1 = 1/m_5 < 0 \), \( \beta_2 = -m_1/m_5 < 0 \), \( \beta_3 = -m_2/m_5 > 0 \), \( \beta_4 = -m_3/m_5 < 0 \), \( \beta_5 = -m_4/m_5 k_0 < 0 \), \( \beta_6 = -m_4/m_5 k_1 < 0 \), \( \beta_7 = -m_4/m_5 k_2 < 0 \), \( \beta_8 = -m_4/m_5 k_3 < 0 \), \( \beta_9 = -m_6/m_5 < 0 \), \( \beta_{10} = -m_7/m_5 k_0^* < 0 \) and \( \beta_{11} = -m_7/m_5 k_1^* < 0 \).

Equation (8) is a long-run real exchange rate relationship. We also added an error term which is assumed to be white noise. According to this equation, a higher supply of money and interest rate results in a lower real exchange rate over the long run. One possible explanation is that a higher money supply or a higher interest rate could cause a higher price over the long run which results in a lower real exchange rate. A higher real income results in a higher real exchange rate over the long run. A higher real income could result in a higher real demand for money and a lower price level and so a higher exchange rate.

A higher government expenditure results in a lower demand for money and, therefore, a higher demand for goods and services and so a higher price level and a lower real exchange rate. This is also true for deficit, outstanding government debt and debt financed externally. A higher commodity price leads to a higher demand for domestic currency and, consequently, it results in a higher value of the Canadian currency (a lower real exchange rate). A higher foreign interest rate leads to a lower demand for money which in turn results in a higher demand for goods and services. The latter demand leads to a higher price level and, therefore, a lower real exchange rate. Factors which affect risk associated with holding foreign currency, i.e., debt and foreign
debt financing of the foreign debt, also lead to a lower real exchange rate. This could be due to
the fact that these factors reduce demand for foreign money ($US). It should be emphasised that
the long-run Equation (8) is also subject to a short-run dynamics of the system which includes
stationary variables which represent crisis as well as policy regime changes and other exogenous
factors which affect the system in both domestic and foreign countries.

3. Data and Long-Run Results

3.1 Data

The model will be estimated on quarterly Canadian data from 1972Q1 to 2010Q3. The
choice of the sample period is based on the time when Canada fully adopted a flexible exchange
rate regime and the fact that the commodity price is available only from 1972.\(^3\) Canada was
chosen because it has the highest degree of openness among the G7 countries, see Table 1.
Furthermore, since the major Canadian trade partner is the United States the Canada-US real
exchange rate was chosen. For example, the average percentage of the sum of Canadian exports
and imports to and from the United States per the total sum of Canadian exports and imports for
the period 1986-2009 was 72.10%. This percentage was 72.23% in 1986 and increased to
77.48% in 1999 and fell to 63.01% in 2009. The sum of U.S. direct and other investments as well
as U.S. portfolio in Canada represented 57% of the Canadian GDP in 2009.

Table 1 about here

The Canadian data was obtained from Statistics Canada CANSIM database. The U.S.
data was obtained from the St. Louis Federal Reserve Database (FRED). Some of the missing
data for both countries were taken from the *International Financial Statistics* (IFS) online.

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\(^3\) After going for a Floating Exchange Rate in the 1950s, Canada again turned to a Fixed Exchange Rate in 1962.
Then it decided to return to a floating rate in June 1970.
On March 22, 2002, the monetary aggregates were adjusted historically to take into account the Canadian Imperial Bank of Commerce (CIBC)'s recent acquisition of the retail client business of Merrill Lynch Canada. M1 used in this paper is March 22, 2002 adjusted data.

To investigate the stationarity property of the variables, I used Augmented Dickey-Fuller and non-parametric Phillips-Perron tests. Furthermore, to allow for the possibility of a break in intercept and slope, I also used tests developed by Perron (1997) and Zivot and Andrews (1992). According to the test results, all variables are integrated of degree one (non-stationary). They are first-difference stationary. It should, however, be noted that the changes of variable debt per GDP for both US and Canada are stationary at only 90% level of significance according to Zivot and Andrews’ (1992) test result. For the sake of brevity, these results are not reported, but are available upon request. All data, when appropriate, is seasonally adjusted. U.S. interest rates were adjusted to 365-day basis. Both Canadian and U.S. variables, when appropriate, are in millions of dollars.

3.2 Long-Run Methodology and Results

Since all variables in Equation (8) have a unit root, we will first verify if a long-run relationship exists between the level of the real exchange and its determinants, as specified by the model.

Table 2 about here

Table 2 reports the cointegration test results on the model. During the sample period, there have been some policy regime changes and/or exogenous changes which could influence the short-run dynamics of the system. As evidenced by Kia (2006b), constant models can have time-varying coefficients if a deeper set of constant parameters characterizes the data generation process. Specifically, the existence of constancy may depend on whether raw coefficients or
underlying parameters are evaluated. Kia (2006b) also shows that the estimated long-run relationship can be biased when the appropriate policy regime changes and/or other exogenous shocks are not incorporated in the short-run dynamics of the system. None of the studies on the real exchange rate so far, to the best of my knowledge, has incorporated this important fact.

To account for those policy regime changes and other exogenous factors which could affect the short-run dynamics of the system, the following dummy variables were constructed. Inftar
(=1 for 1991, first quarter and after, and zero otherwise)
, Free
(=1 since 1991, first quarter and zero otherwise)
, and Nafta (=1 since 1994, first quarter and zero otherwise)
, and crisis (=1 since 2007, fourth quarter and after and zero otherwise)
. For more on the first three dummy variables see Kia (2006b).

Equation (8) was estimated by allowing the estimate of the short-run dynamics of the equation to be affected by policy regime changes represented by the above dummy variables. In determining the lag length, one should verify if the lag length is sufficient to get white noise

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4 This dummy variable accounts for the introduction of inflation rate targeting by the Department of Finance and the Bank of Canada in February 1991. Clearly, the reduction of inflation uncertainty increases services of money.

5 This dummy variable accounts for the implementation of the Free Trade Agreement between Canada and the United States in January 1991.

6 This dummy variable accounts for the implementation of the North American Free Trade Agreement (NAFTA) between Canada, the United States and Mexico in January 1994. Both free trade and NAFTA agreements could improve the services of money by allowing the holder of Canadian dollars to purchase goods and services produced outside Canada at the same price, excluding transportation cost, charged at the production site.

7 This dummy variable accounts for the current financial crisis which started in the last quarter of 2007.

8 Note that Equation (8) is a valid relationship only if the PPP between Canada and the US does not hold. I tested for PPP by investigating if a long-run cointegrating relationship among $E_t$, $p_t^*$ and $p_t$ exists. Specifically, I tested for the existence of a cointegrating vector in $\log(p_t) = \log(p_t^*) + \log(E_t)$ relationship while allowing the short-run dynamics of the system to adjust for policy regime changes and other exogenous shocks which could affect the relationship over the short run. I failed to find any long-run relationship, implying that PPP does not hold over the long run between Canada and the US. Consequently, Equation (8) should hold. For the sake of brevity, the test result is not reported, but is available upon request.
residuals. As it was recommended by Hansen and Juselius (1995, p. 26), set \( p=r \) (the unrestricted model) in Equation (8) and test for autocorrelation and heteroskedasticity. In this case the residuals are the OLS-estimates from Model (8). LM tests will be employed to confirm the choice of lag length. The order of cointegration \( (r) \) will be determined by using the Trace test developed in Johansen and Juselius (1991). Since we allow the short-run dynamics of the system to be affected by the dummy variables accounting for policy regime changes and the recent financial crisis we need to simulate the critical values as well as their associated \( p \)-values for the rank test. The CATS in RATS computer package (Version 2, see Dennis, 2006) was used to simulate the critical values. The number of replications is 2500 and the length of random walk is 400. Bartlett correction factors of the rank test statistic derived in Johansen (2000 and 2002) were used to correct for a potential bias possibly generated by small sample errors. A four-lag length was needed in order to ensure that the error term is white noise.

The only non-congruency is non-normality. However, as it was mentioned by Johansen (1995), a departure from normality is not very serious in cointegration tests. According to the result of Table 2, Trace tests reject \( r=0 \) at the 95% level while we cannot reject \( r\leq1 \), implying that \( r=1 \). Figures 1 to 3 plot the calculated values of the recursive test statistics for the long-run relationship. Note that all recursive tests are normalized by the 5% critical value. Thus, calculated statistics that exceed unity imply the rejection of the null hypothesis and suggest unstable cointegrating vectors. The curve \( X(t) \) plots the actual disequilibrium as a function of all short-run dynamics including dummy variables, while the \( R1(t) \) curve plots the “clean” disequilibrium that corrects for short-run effects. We hold up the first 25 years for the initial estimation. As these figures show, all these equations appear stable over the long run and the LR
tests are also stable when the models are corrected for short-run effects. Having established that our cointegration relationship is stable, we will analyze this relationship.

Figures 1 to 3 about here

The bottom panel in Table 2 reports the estimate of the long-run real exchange rate determination. The estimated coefficients of the determinants of the real exchange rate, except for deficit per GDP, foreign financed debt per GDP and the US debt per GDP, in the cointegrating space are as our theoretical model, Equation (8), indicates.

(i) Monetary policy: According to our theoretical model, Equation (8), we would expect both the level of money supply and interest rate to have a negative influence on the real exchange rate over the long run. Based on our estimation result both variables have the correct sign but are statistically insignificant which means they have the same impact on the nominal exchange rate and the price so that the real exchange remains the same.

(ii) Fiscal policy: The long-run estimated coefficient of the log of real government expenditures is negative, as our model predicts, and statistically significant. The long-run estimated coefficient of deficit per GDP is positive and statistically significant. This result contradicts our theoretical model. One possible explanation is that a higher deficit per GDP over the long run may be considered an expected accumulation of government debt and leads to the devaluation of the domestic currency. The estimated coefficient of government debt per GDP is statistically significant and has a negative sign as the model predicts. The estimated coefficient of externally-financed government debt per GDP is not statistically significant over the long run and has a wrong sign.

(iii) The commodity price: The estimated coefficient of the commodity price is negative (confirms the theoretical model) and statistically significant.
(iv) External factors: Both the estimated coefficient of US interest rate and US foreign-financed debt per GDP have the correct sign and are statistically significant, but the estimated coefficient of the US outstanding debt per GDP, while statistically significant, has a wrong sign. Specifically, its impact on the real exchange rate in Canada is positive over the long run. One possible explanation is that as debt goes up in the United States there will be an expected monetization of the debt and, therefore, a higher price in the future. This may result in a higher US price today and so a higher real exchange rate.

4. Short-Run Dynamic Models

Having established in the previous section that a long run relationship describing the real exchange rate and its determinants exists, we need to specify the ECMs (error correction models) that are implied by our cointegrating vector. Following Granger (1986), we should note that if small equilibrium errors can be ignored, while reacting substantially to large ones, the error correcting equation is non linear. All possible kinds of non linear specifications, i.e., squared, cubed and fourth powered of the equilibrium errors (with statistically significant coefficients) as well as the products of those significant equilibrium errors were included.

To avoid biased results we allow for a lag profile of eight quarters. Furthermore, since having too many coefficients can lead to inefficient estimates we ensure parsimonious estimation by selecting the final ECMs on the basis of Hendry’s General-to-Specific approach. Assuming government spending and foreign-financed debt per GDP (over the short run) as well as foreign variables are exogenous, we will have seven endogenous variables in the system. Consequently, we have seven error-correction models. For the sake of brevity, I only report the parsimonious ECM for the growth real exchange rate. Other results are available upon request. However, the full estimation results of all these ECMs will be used to analyze the unanticipated shocks in
endogenous variables using impulse response functions. Table 3 reports the parsimonious results from estimating the ECM.

In this table $\Delta$ denotes a first difference operator and EC, $\overline{R^2}$, $\sigma$ and DW, respectively, denote the error correction term from long-run equation for the real exchange rate, the adjusted squared multiple correlation coefficient, the residual standard deviation and the Durbin-Watson statistic. White is White’s (1980) general test for heteroskedasticity, ARCH is five-order Engle’s (1982) test, Godfrey is five-order Godfrey’s (1978) test, REST is Ramsey’s (1969) misspecification test, Normality is Jarque-Bera’s (1987) normality statistic, $L_1$ is Hansen’s (1992) stability test for the null hypothesis that the estimated $i$th coefficient or variance of the error term is constant and $L_c$ is Hansen’s (1992) stability test for the null hypothesis that the estimated coefficients as well as the error variance are jointly constant.

According to the specification results reported in Table 3, none of these diagnostic checks is significant. Based on Hansen’s stability test result, all of the coefficients, individually or jointly, are stable. However, the variance of the estimate is not stable. The estimation method is OLS. The level of the dummy variables describing policy regime shifts and exogenous changes as well as a linear time trend were included in the EC model. As it was mentioned in the previous section, these dummy variables also appeared in the short-run dynamics of the system in our cointegration regression.

As we can see none of the domestic fiscal variables has any impact, over the short run, on the growth of the real exchange rate. According to our estimation results reported in Table 3, the estimated coefficient of the error-correction term is negative and statistically significant. Interestingly, the impact of the equilibrium error is non-linear implying that agents may ignore
the small deviation from the equilibrium in currency exchange and/or goods markets, but react drastically to a large disequilibrium by deviating more from equilibrium since the estimated coefficient of the squared EC is positive. The impact of the interest rate as it was also expected by the model is negative, but contrary to its long-run effect it is statistically significant. Specifically, the change in the interest rate results in a reduction in the growth of the real exchange rate.

The estimated coefficient of the change in the US debt per GDP is negative as it is expected theoretically, implying that, over the short run, an increase in the US debt per GDP, everything else being equal, leads to an inflow of capital and the depreciation of the exchange rate (appreciation of Canadian dollar) and, therefore, a reduction in the real exchange rate.

5. Impulse Responses and Real Exchange Rate

To analyze the impact of shocks in domestic factors to the real exchange rate, I use the estimated coefficients of all ECMs by considering the associated impulse responses. In order for each variable to be independently shocked I use Choleski factor to normalize the system so that the transformed innovation covariance matrix is diagonal. The conclusions are potentially sensitive to the ordering (or normalization) of the variables. As one would expect, part of a shock in the government expenditures is contemporaneously correlated to a shock in deficit, outstanding debt and debt financing as well as money supply which by themselves are correlated to a shock in domestic interest rate, in commodity price, in GDP and in the real exchange rate. Consequently, let us propose the ordering of lrg, defgdp, debtgdp, fdgdp, lrm1, i, lcom, ly and lq.

By ordering the real exchange rate at the last, the identifying restriction is that the other variables do not respond contemporaneously to a shock to the real exchange rate. This ordering
is not critical in our analysis as, to the best knowledge of the author, no particular theory or empirical evidence conflicts with the logic of the proposed ordering.

The VAR was run in the error-correction form with four lags (the lag length of the cointegration equations, see Table 2). The impulse response functions reflect the implied response of the levels. US interest rate, US debt per GDP and US foreign-financed debt per GDP are included as exogenous variables. Other deterministic variables include dummy variables which account for policy regime changes or other exogenous shocks.

Let us follow Lütkepohl and Reimers (1992) and assume a one-time impulse on a variable is transitory if the variable returns to its previous equilibrium value after some periods. If it settles at a different equilibrium value, the effect is called permanent. Figure 4 plots the impulse responses of the real exchange rate to a shock in \( lg, \text{defgdp, debtgdp, fdgdp, lrm1, i, ly, lcom and lq.} \) Since, in computing confidence bands, neither the coefficients of VAR nor their responses to shocks are known with certainty, the Monte Carlo simulation is used. The number of Monte Carlo draws is 1000. For the sake of brevity, we only concentrate on the impulse responses of \( lq \) to a shock on other variables. As we can see in the figure all responses are within the confidence band.

Figure 4 about here

There are four noteworthy features of the impulse responses. First, all impulse responses, except money supply and real income, are permanent. Specifically, according to Plot E, a one standard deviation shock to real money supply (equal to 0.02 units) induces a contemporaneous increase of 0.0046 units in the real exchange rate and continues to 0.0052 in the 10\(^{th}\) quarter before falling in magnitude and reaches, e.g., to 0.0036 units at the 24\(^{th}\) quarter; therefore, the impulse is temporary. Furthermore, a one standard deviation shock to real GDP induces a
contemporaneous (Plot G) fall in the real exchange rate by 0.003 units but the decline reduces in magnitude to 0.0005 in 24th quarter. Second, a monetary policy shock, by changing interest rate, results in an appreciation of the real Canadian dollar permanently (Plot F). Third, as for the fiscal variables, one standard deviation shock to real government expenditure and debt financed externally reduces permanently the real exchange rate, but the same shock to the deficit and debt per GDP will increase permanently the real exchange rate (plots A to D). Finally, a one standard deviation shock to the commodity price results in a permanent real appreciation in the real Canadian dollar, i.e., a permanent fall in the real exchange rate (Plot H).

To gauge whether fiscal, monetary and other shocks have played much of a role in accounting for movements in the real exchange rate, we analyze variance decompositions for various time horizons (see Table 4). Each row shows the fraction of the t-step ahead forecast error variance for the real exchange rate that is attributed to shocks to the column variables. According to these results, domestic interest rate and real government expenditure account for an insignificant percentage of the real exchange rate forecast error variance at all horizons. The deficit per GDP, debt per GDP, foreign-financed debt per GDP and commodity price index shocks account for an increasing percentage of the real exchange rate forecast error variance as the time horizon increases. However, the stronger contributor to forecast error variance is the debt per GDP. For example, after a year its shocks account for 9.50% of the real exchange rate forecast error variance and it rises to 20.11% after twenty four quarters. Finally, real money supply and real GDP shocks have a decreasing percentage of the real exchange rate forecast error variance as the time horizon increases.

Table 4 about here
6. Concluding Remarks

In order we can use the real exchange rate as a measure of price-cost competitiveness we need to know its fundamental determinants. I developed a monetary model of the real exchange rate in this paper and showed that long-run determinants of the real exchange rate are a function of the real money supply, domestic and foreign interest rate, real GDP, real government expenditure, deficit per GDP, domestic and foreign outstanding debt per GDP, domestic and foreign externally-financed debt per GDP and the commodity price. The model was tested on Canadian data for the period 1972-2010. Canada has the highest degree of openness among G7 countries and, therefore, provided good grounds for testing the model.

In fact, it was found the model holds over the long run, i.e., there is a cointegration relationship in space. It was found that the long-run estimated coefficient of all variables, except deficit per GDP, foreign-financed debt per GDP and the US debt per GDP, in the long-run real exchange rate has the correct sign according to the long-run Equation (8). Furthermore, these coefficients, except for domestic interest rate, foreign-financed debt per GDP and US interest rate, are statistically significant. However, it was found that over the short-run period of the domestic fiscal variables has any statistically significant impact on the growth of the real exchange rate.

Monetary variables, i.e., the change in interest rate and the growth of money supply, have a negative impact on the growth of the real exchange rate as it was expected theoretically. Importantly, it was found that the agents ignore a small deviation from equilibrium in the exchange market while reacting drastically to a large deviation.

Moreover, the impulse responses of the real exchange rate to all shocks, except money supply and real income, are permanent. The real exchange rate appreciates for a shock to the
interest rate, deficit and debt per GDP and commodity price while it depreciates for a shock to the real government expenditure and foreign-financed debt per GDP. Deficit, debt and foreign-financed debt per GDP as well as commodity price index shocks account for an increasing percentage of the real exchange rate forecast error variance as the time horizon increases. However, the stronger contributor to forecast error variance is the commodity price index.
References


Table 1(1): Degree of Openness of the G7 Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>.460460</td>
<td>.060791</td>
</tr>
<tr>
<td>Germany</td>
<td>.598879</td>
<td>.118770</td>
</tr>
<tr>
<td>Canada</td>
<td>.628150</td>
<td>.122911</td>
</tr>
<tr>
<td>France</td>
<td>.472049</td>
<td>.050400</td>
</tr>
<tr>
<td>Japan</td>
<td>.225167</td>
<td>.051296</td>
</tr>
<tr>
<td>U. K.</td>
<td>.537810</td>
<td>.034240</td>
</tr>
<tr>
<td>U. S.</td>
<td>.218507</td>
<td>.035463</td>
</tr>
</tbody>
</table>

(1) The degree of openness is measured by: (Imports + Exports)/GDP
All data is in the country’s own currency, except the data for Italy, Germany, and France which since 1999 are in Euros. The Source of data is IFS database.

Table 2(1): Long-Run Test Result
Tests of the Cointegration Rank

<table>
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<tr>
<th>H₀= r</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Diagnostic tests</th>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>p-value</td>
</tr>
<tr>
<td>Trace</td>
<td>340.07</td>
<td>243.90*</td>
<td>178.25</td>
<td>124.20</td>
<td>81.48</td>
<td>39.23</td>
<td>21.47</td>
<td>7.21</td>
<td>1.02</td>
<td>Autocorrelation</td>
</tr>
<tr>
<td></td>
<td>Trace 95</td>
<td>295.66</td>
<td>248.29</td>
<td>204.12</td>
<td>165.98</td>
<td>130.70</td>
<td>97.61</td>
<td>70.00</td>
<td>45.42</td>
<td>23.02</td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Lag length = 4</td>
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<tr>
<td>p-value</td>
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<td>0.08</td>
<td>0.91</td>
<td>0.39</td>
<td>0.81</td>
<td>0.96</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

Normalized

<table>
<thead>
<tr>
<th>Normalized</th>
<th>liq</th>
<th>lrml</th>
<th>i</th>
<th>ly</th>
<th>lrg</th>
<th>defgdgp</th>
<th>debtgdp</th>
<th>fdgdgp</th>
<th>lcom</th>
<th>i*</th>
<th>debtgdp*</th>
<th>fdgdgp*</th>
<th>Con</th>
</tr>
</thead>
<tbody>
<tr>
<td>liq</td>
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<td>-0.24</td>
<td>-0.16</td>
<td>4.87</td>
<td>-6.21</td>
<td>25.07</td>
<td>-2.51</td>
<td>1.94</td>
<td>-0.67</td>
<td>-0.01</td>
<td>5.18</td>
<td>-8.47</td>
<td>-2.78</td>
</tr>
<tr>
<td>(t-statistics)</td>
<td>(-1.26)</td>
<td>(-1.67)</td>
<td>(6.99)</td>
<td>(-7.64)</td>
<td>(10.79)</td>
<td>(-2.54)</td>
<td>(10.85)</td>
<td>(-4.03)</td>
<td>(-0.15)</td>
<td>(4.34)</td>
<td>(-4.66)</td>
<td>(-1.37)</td>
<td></td>
</tr>
</tbody>
</table>

(1) a = means accept the null of r=1.

(2) By using Bartlett correction the Trace test statistic is corrected for the small sample error, see Johansen (2000 and 2002)
(3) Because of the inclusion of the dummies in the short-run dynamics of the system the limit distribution of the rank statistics should be simulated. The CATS 2 in RATS computer package was used to simulate the critical values. The number of replications was 2500 with a length of random walks of 400.
(4) The approximate p-value using the corrected test statistic. LM(1) and LM(2) are one and two-order Lagrangian Multiplier test, respectively.

The sample period is 1972Q1-2010Q3. liq is the log of the real exchange rate, where q is defined as E p*/p, E is the nominal exchange rate (domestic price of US currency), p* and p are the US and Canadian price levels (CPI), respectively, lrml is the log of real M1 and i is the log[R/(1+R)], where R is three-month TB rate in decimal points. liq is the log of the real GDP. lrg is the log of real government expenditures on goods and services, defgdgp and debtgdp are deficit and outstanding debt per GDP, respectively. fdgdgp is the amount of foreign financed debt per GDP and lcom is the log of commodity price. i* = log(R*/1+ R*), where R* is the US three-month TB rate in decimal points. debtgdp* is the US outstanding debt per GDP, fdgdgp* is the amount of foreign financed US debt per GDP and Con is the constant term.
Table 3\(^{(1)}\): Error Correction Model for the Growth of the Real Exchange Rate

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Hansen’s (1992) stability L(_i) test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta lq)(_{t-2})</td>
<td>-0.15</td>
<td>0.07</td>
<td>0.94</td>
</tr>
<tr>
<td>(\Delta i)(_{t-3})</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.37</td>
</tr>
<tr>
<td>(\Delta lrm)(_{t-2})</td>
<td>-0.27</td>
<td>0.09</td>
<td>0.56</td>
</tr>
<tr>
<td>(\Delta ly)(_{t-1})</td>
<td>-0.62</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>(\Delta lcom)(_{t-3})</td>
<td>-0.10</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td>(\Delta lcom)(_{t-4})</td>
<td>-0.10</td>
<td>0.04</td>
<td>0.75</td>
</tr>
<tr>
<td>(\Delta debtgdp)(_{t-2})*</td>
<td>-0.93</td>
<td>0.28</td>
<td>0.26</td>
</tr>
<tr>
<td>(\Delta debtgdp)(_{t-3})*</td>
<td>-1.44</td>
<td>0.31</td>
<td>0.03</td>
</tr>
<tr>
<td>(EC)(<em>t)(</em>{4})</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.38</td>
</tr>
<tr>
<td>((EC)^{2})(_{t-2})</td>
<td>0.03</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td>crisis</td>
<td>0.02</td>
<td>0.01</td>
<td>1.00</td>
</tr>
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</table>

Hansen’s (1992) stability L\(_i\) test on variance of the ECM = 0.00
Joint (coefficients and the error variance) Hansen’s (1992) stability L\(_i\) test = 0.04

(1) The sample period is 1972Q1-2010Q3. \(\Delta\) means the first difference, Mean of dependent variable=-0.0001. EC is the error correction term from the cointegration vector. For the definition of the other variables see footnote to Table 2. The estimation method is the OLS. \(R^2=0.34\), \(\sigma=0.03\), DW=2.03, Godfrey(6)=0.41 (significance level=0.97), White=74.93 (significance level=0.55), ARCH(5)=5.59 (significance level=0.34), RESET(3)=1.29 (significance level=0.28) and Normality(\(\chi^2=2\))=0.29 (significance level=0.86).

To mitigate non-normality of the disturbance term in the first round of estimation, I included dummy variables accounting for the outlier observed in 2008Q4. This dummy variable has a value of one in the fourth quarter of 2008, and zero otherwise. The estimated coefficient of this dummy variable is not reported, but is available upon request.
<table>
<thead>
<tr>
<th>Period (Quarters)</th>
<th>lq</th>
<th>lrml</th>
<th>i</th>
<th>ly</th>
<th>lrg</th>
<th>defgdp</th>
<th>debtgdp</th>
<th>fdgdp</th>
<th>lcom</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>82.39</td>
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<td>0.95</td>
<td>1.80</td>
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<td>0.90</td>
<td>1.30</td>
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<tr>
<td>4</td>
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<td>9.50</td>
<td>4.47</td>
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<tr>
<td>12</td>
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<td>5.34</td>
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<td>20</td>
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<td>24</td>
<td>46.53</td>
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<td>11.64</td>
<td>20.11</td>
<td>5.91</td>
<td>12.17</td>
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* See footnote of Table 2 for the definition of mnemonics.
Figure 1: Recursive Likelihood Ratio Tests*

Test of Beta Constancy

\[ Q(t) \]

5% C.V. (3.7 = Index)

\[ X(t) = \text{the actual disequilibrium as a function of all short-run dynamics and dummy variables.} \]
\[ R1(t) = \text{the “clean” disequilibrium that corrects for short-run effects.} \]

Figure 2: Test for Constancy of the Parameters of the Model

Beta 1 (R1-model)
Figure 3: Test for the Hypothesis that Betas of Each Sub-Period Equal to Betas of the Entire Sample of Model*

Test of Beta(t) = 'Known Beta'

*X(t) = the actual disequilibrium as a function of all short-run dynamics and dummy variables. R1(t) = the "clean" disequilibrium that corrects for short-run effects.
Figure 4: Impulse Responses of the Real Exchange Rate to a Shock to Other Variables

Plot A
Responses to Real Government Expenditure

Plot B
Responses to Deficit per GDP

Plot C
Responses to Debt per GDP
Figure 4 Continues
Plot D
Responses to Foreign-Financed Debt Per GDP

Plot E
Responses to Real M1

Plot F
Responses to Domestic Interest Rate
Figure 4 Continues

Plot G
Responses to Real GDP

Plot H
Responses to Real Commodity Price

Plot I
Responses to the Real Exchange Rate